Optimal Carbon Tax with Dynamic Skill Heterogeneity

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Abstract

Extending a dynamic Mirrlees model to two goods, one polluting, this paper studies optimal carbon taxation with redistribution. I find that carbon taxes should be nonlinear and vary with age when labor is distorted and consumption goods differ in their labor complementarity. The social cost of carbon also reflects the effect of pollution on work incentives. When labor and savings wedges are fixed, carbon taxation can partially offset these constraints. In a US calibration, when income taxes can be adjusted, I find that the best linear carbon tax captures 99% of the welfare gains of a fully nonlinear carbon tax. When income wedges are fixed at US-calibrated levels, age-dependent carbon taxes capture 87% of the welfare gain from moving from linear to fully nonlinear carbon taxation. However, these gains are small compared with those of modest income tax reforms, suggesting that greater flexibility in carbon pricing is a poor substitute for income tax reform.

Keywords: Carbon tax, dynamic Mirrlees taxation, nonlinear commodity taxation, income redistribution, social cost of carbon
JEL Classification: H21; H23; D82; Q58; D62

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1 Introduction

If the ability-to-pay principle motivates progressive income taxation, should it likewise call for progressive carbon taxation? The standard answer is that poor and rich alike should face the same carbon tax with income taxation alone handling redistribution, yet some economists have proposed nonlinear carbon taxes to address unequal emissions or imperfect income taxes (Chancel and Piketty 2015; Stiglitz 2019; Fleurbaey and Kornek 2021; Chancel 2022). How should a government that cares about redistribution design a carbon tax and how well can a nonlinear carbon tax address income tax imperfections?

In this paper, I extend the dynamic Mirrlees income taxation model of Golosov et al. (2016) to a two-good setting where one good generates pollution and characterize the wedges of an optimal allocation. I find that the consumption wedge between the "clean" and the "dirty" good depends on the household's entire skill history, implying a nonlinear carbon tax that varies with age, when labor is distorted and one good is more complementary with labor. Moreover, each term in the sum of discounted marginal damages in the social cost of carbon is inflated when labor is distorted and pollution discourages work, increasing the carbon tax. When labor is additively separable from both consumption goods and pollution in preferences, the carbon tax is equal to the standard Pigouvian level. These results extend the consumption taxation results of Mirrlees (1976) and carbon taxation results of Jacobs and de Mooij (2015) to a dynamic setting, showing that consumption taxes should be age-varying and that the social cost of carbon should be adjusted to account for lifetime work incentives.

I also study optimal allocations when labor and savings wedges are held fixed at prescribed schedules, representing a government that designs a carbon tax without reoptimizing the income tax system. The welfare criterion is the same as for the jointly optimal allocation and only the set of feasible allocation shrinks, so this exercise measures the possible welfare gain from a more flexible carbon tax instrument when the income tax in force differs from the optimum under that criterion[1]. I characterize the optimal consumption wedge, which now has terms that may partially offset the labor and savings wedge constraints. The size and direction of the effect depend on how the willingness to pay for the dirty good evolves with income and clean good consumption. Whereas the standard dynamic Mirrlees framework considers jointly optimal allocations, this paper is to my knowledge the first to study optimal dynamic nonlinear consumption wedges when both labor and savings wedges are constrained.

[1]Note that I am not making any judgments on the optimality of the tax system and only use it as a benchmark when the income tax system is not one the planner would prefer, regardless of the reasons this might be the case. In theory, observed income tax schedules could be rationalized as optimal under a different welfare criterion, in which case the fixed schedules will just so happen to coincide with the planner's preferences, yielding the same theoretical and numerical results as the unconstrained problem.

To get an idea of the quantitative importance of the results, I calibrate the model to US data and compare the welfare gains from relaxing restrictions on carbon and income taxes. I find that a linear carbon tax captures 99.5% of the gain from introducing fully nonlinear carbon taxes when income taxes are jointly optimized. When income taxes are fixed at the calibrated shcedules, fully nonlinear carbon taxation improves welfare over the linear carbon tax, with 86.5% of the additional gain captured by age-dependence alone. This carbon tax starts above the social cost of carbon and falls with age, most of this decline and additional progressivity coming from a fixed overdistorted savings wedge in the calibration. However, reoptimizing income taxes remains a better option than making the carbon tax more flexible, yielding about 17 times the gain from moving to a fully nonlinear carbon tax. In the calibration, reducing capital income tax rates by 0.9 percentage points or labor income tax rates by 1.2 percentage points delivers about the same welfare gain as moving from a linear to an age-dependent carbon tax, so a more flexible carbon pricing instrument is a poor substitute for income tax reform. If income tax reform is unavailable however, age-dependent carbon taxes are a good alternative to full nonlinearity, which can be hard to implement. This finding on dynamic consumption taxation echoes that of Weinzierl (2011) and Farhi and Werning (2013) on income taxation, who likewise show that optimal nonlinear income taxes can be well-approximated with age-dependent income taxation.

My work connects to the literature on optimal income and consumption taxes. Mirrlees (1976) extends the analysis in Mirrlees (1971) to study the optimal taxation of goods, finding that relative leisure complements should be taxed more heavily to discourage their consumption and indirectly encourage work, helping redistribution. Atkinson and Stiglitz (1976) find that, when preferences are weakly separable in consumption and labor, there is no motive for differentiated consumption taxes. In a static setting with non-separable preferences, Jacobs and Boadway (2014) further show that implementing the optimal allocation generally requires nonlinear consumption taxes. Golosov et al. (2003) extend the Atkinson-Stiglitz result to a dynamic setting with weakly separable preferences. This paper extends the Mirrlees (1976) results on consumption taxation to a dynamic setting without assuming weakly separable preferences, showing that the resulting consumption wedge is not only nonlinear but also history-dependent.

When consumption generates pollution, consumption taxes gain an externality correction role. Cremer et al. (1998) find that the optimal marginal tax rate on the dirty good combines a corrective term and an incentive term. Jacobs and de Mooij (2015) show that the social value of reducing pollution can also depend on how environmental quality interacts with labor supply. When income and consumption taxes are jointly optimal and allowed to be nonlinear, they also show that the labor wedge keeps the standard structure in Mirrlees (1971) and Saez (2001), with no additional externality correction terms. When

consumption goods and environmental quality are weakly separable from labor in the utility function, their carbon tax is equal to the social cost of carbon. I extend their analysis to a dynamic setting with persistent pollution and income risk, showing that the externality introduces no additional terms in income wedges.

A parallel literature studies carbon taxation alongside income taxation using the Ramsey approach with linear tax instruments. In a setting where the government uses linear consumption taxes to fund its expenditures, Sandmo (1975) finds that the optimal tax on the dirty good is the sum of an externality correction term and a revenue-raising Ramsey term, whereas the tax formulas for the other goods keep their structure. Bovenberg and de Mooij (1994) and Bovenberg and Goulder (1996) find that the carbon tax should be lower than the social cost of carbon because it discourages labor supply, even if its revenue finances labor income tax cuts.

Barrage (2020) extends this static analysis to a dynamic setting, putting Golosov et al. (2014) in a Ramsey taxation framework. She finds an optimal formula for the carbon tax, summing production damages and utility damages divided by the marginal cost of public funds. Her calibrated model suggests the optimal carbon tax should be 8% below the first-best carbon tax. Douenne et al. (2026) and Douenne et al. (2025) extend this analysis to a heterogeneous-agent setting and find that the optimal carbon tax is nearly Pigouvian.

When income taxes cannot be adjusted, carbon taxation may help offset distortions elsewhere in the tax system. Fleurbaey and Kornek (2021) show that nonlinear corrective taxes can improve welfare when income taxes are fixed, even when preferences are separable between consumption and labor. In an economy with uninsurable income risk, Wöhrmüller (2025) finds that a uniform carbon tax combined with lump-sum transfers provides additional insurance when labor income taxes are fixed. Barrage (2020) finds that the optimal carbon tax is lower than the first-best level when labor or capital income taxes are fixed. Douenne et al. (2026) find that fixing the labor income tax lowers the carbon tax and fixing the capital income tax raises it. In a setting with uninsurable income risk, Douenne et al. (2025) find that optimal carbon taxation remains close to the social cost of carbon even when income tax rates are fixed at observed levels. These quantitative studies impose carbon tax rates that are common across households at each date. I also allow consumption wedges to vary across household skill histories.

Section 2 presents the lifecycle economy. Section 3 characterizes the optimal labor, savings, and consumption wedges. Section 4 characterizes the consumption wedge when labor and savings wedges are fixed. Section 5 compares the welfare gains from relaxing restrictions on carbon and income taxation, and Section 6 concludes.

2 Environment

The economy is a two-good extension of the dynamic Mirrlees setting of Golosov et al. (2016), where consumption of one of the goods contributes to a persistent pollution stock harming utility. Households face privately observed skill shocks throughout their lives and a social planner allocates consumption and labor to maximize social welfare, providing redistribution and insurance while correcting the externality. I consider a single country and a single cohort to abstract from international and intergenerational equity concerns, so time and age coincide in this model[2].

2.1 Households and information

There is a unit mass of atomistic households living at dates $t = 0, \dots, T$. A household with skill $\theta_t > 0$ supplies labor l_t and earns labor income $y_t = \theta_t l_t$. Initial skill is drawn from a distribution F_0 with density f_0 and subsequent skills follow a Markov process $F_t(\theta|\theta_{t-1})$ with density f_t. Households are differentiated by their skill history, $\theta^t = (\theta_0, \dots, \theta_t)$, with Θ^t the set of all possible skill histories up to date t.

Assumption 2.1 (Skill process)**.**
For $t = 1, \dots, T$, the transition density $f_t(x|s)$ defined on the domain $(0, \infty)$ is differentiable in the previous period's skill. Write $f_{2,t} \equiv \partial f_t / \partial s$. For every compact interval $I \subset (0, \infty)$, there exists $m_{t,I} \in L^1(0, \infty)$ such that

$$\sup_{s \in I} |f_{2,t}(x \mid s)| \leq m_{t,I}(x) \qquad \text{a.e} \quad x > 0$$

Each household consumes a clean good c_t and a dirty good q_t, whose aggregate Q_t generates emissions contributing to a pollution stock S_t affecting utility. Expected lifetime utility is

$$\mathbb{E}_0 \left[\sum_{t=0}^{T} \beta^t U(c_t, q_t, l_t, S_t) \right]$$

where $\beta \in (0, 1)$ is the discount factor. Both consumption goods increase utility, while labor and pollution decrease it. I allow non-separability in these arguments, meaning consumption and pollution can affect each other's marginal utility as well as that of labor. Households are atomistic, taking the pollution stock as given.

The planner observes each household's labor income and consumption of both goods, but

[2]This can also be read as a cross-section in a given year when there are no transfers between cohorts and the social cost of carbon is fixed at a given level, which will be the interpretation adopted for the quantitative exercise to yield more interpretable results.

neither skill nor labor supply[3]. A high-skill household can therefore earn as much as a low-skill household by working less, such that the planner cannot distinguish them. This creates an information problem constraining the planner's ability to provide insurance and redistribution.

Preferences satisfy a single crossing condition in labor earnings, meaning higher-skill households require less compensation in the clean good to earn an additional unit of labor income, for given consumption, labor income, and pollution. Let $U_x \equiv \partial U/\partial x$.

Assumption 2.2 (Preferences)**.**
U is twice continuously differentiable, with $U_c, U_q > 0$, $U_l, U_S < 0$, and $U_{cc}, U_{qq}, U_{ll} \leq 0$. For given labor and pollution levels, preferences over goods are strictly quasi-concave with

$$2U_c U_q U_{cq} - U_c^2 U_{qq} - U_q^2 U_{cc} > 0$$

I assume strict single crossing in labor earnings, holding (c, q, y, S) fixed,

$$\frac{\partial}{\partial \theta}\left[\frac{U_l(c, q, y/\theta, S)}{\theta U_c(c, q, y/\theta, S)}\right] > 0$$

2.2 Technology and pollution

Output, which equals labor income, can be converted into either consumption good. The clean good is the numéraire and, for simplicity, both goods have unit resource cost. A linear savings technology transfers resources from one period to the next at a constant gross return $R > 0$. In this model, capital does not enter production.

The clean good generates no emissions, while each unit of the dirty good generates a unit of emissions. Let F^t be the unconditional population distribution of skill histories. Aggregate emissions are

$$Q_t = \int_{\Theta^t} q_t(\theta^t) dF^t(\theta^t) \qquad t = 0, \ldots, T \tag{1}$$

Current and past emissions determine the pollution stock.

$$S_t = \mathcal{S}(S_0, \ldots, S_{t-1}, Q_0, \ldots, Q_t) \tag{2}$$

[3]Intuitively, skill may for example be viewed as talent or another unobservable characteristic. Other aspects of skill could be observable but unusable, such as physical characteristics that society might not allow the planner to act on. Likewise, labor supply may be viewed as effort, which may be hard to track.

$\mathcal{S}$ is differentiable and non-decreasing in each argument. Emissions can persist and affect pollution accumulation at later dates. This equation can also be rewritten as $S_t = \mathcal{S}(\mathcal{H}_t, Q_t)$, where $\mathcal{H}_t \equiv (S_0, \ldots, S_{t-1}, Q_0, \ldots, Q_{t-1})$. I do not otherwise restrict the functional form of $\mathcal{S}$. Pollution harms utility, but not output.

2.3 Planner's problem and wedges

Following the standard approach in the dynamic Mirrlees literature, I use a direct mechanism to characterize the allocations attainable under these information constraints. In simple terms, this essentially means that the planner asks households for their skill and then allocates consumption and earnings according to their report. The planner commits to an allocation rule at date 0 and there is no hidden saving or trading between households. So that the planner's allocation is indeed implemented, the mechanism must make truthful reporting by households optimal, meaning it must be incentive-compatible. By the revelation principle, any equilibrium allocation attainable through another mechanism, such as a tax system, can be reproduced by such a direct mechanism.

An allocation specifies non-negative quantities $c_t(\theta^t)$, $q_t(\theta^t)$, and $y_t(\theta^t)$, which are functions of the household's reported skill history θ^t. For given Pareto weights $\alpha(\theta) \geq 0$ on households with initial skill θ_0, the planner maximizes

$$\int_0^\infty \alpha(\theta)\, \mathbb{E}_0 \left[\sum_{t=0}^{T} \beta^t U \left(c_t(\theta^t), q_t(\theta^t), \frac{y_t(\theta^t)}{\theta_t}, S_t \right) \middle| \theta \right] dF_0(\theta) \tag{3}$$

Feasibility requires that the present value of consumption does not exceed the present value of output plus initial resources, given by initial capital K_0.

$$\int_0^\infty \mathbb{E}_0 \left[\sum_{t=0}^{T} \frac{c_t(\theta^t) + q_t(\theta^t) - y_t(\theta^t)}{R^t} \middle| \theta \right] dF_0(\theta) \leq RK_0 \tag{4}$$

The planner's allocation must be incentive-compatible, meaning that households must weakly prefer to report their true skill. Let $\sigma_t(\theta^t)$ be the report of a household θ^t and let $\sigma^t(\theta^t) = (\sigma_0(\theta^0), \ldots, \sigma_t(\theta^t))$ be its reporting history up to period t. This translates to the following constraint for every household and every reporting strategy.

$$\begin{aligned} &\mathbb{E}_0 \left[\sum_{t=0}^{T} \beta^t U \left(c_t(\theta^t), q_t(\theta^t), \frac{y_t(\theta^t)}{\theta_t}, S_t \right) \middle| \theta \right] \\ &\geq \mathbb{E}_0 \left[\sum_{t=0}^{T} \beta^t U \left(c_t(\sigma^t(\theta^t)), q_t(\sigma^t(\theta^t)), \frac{y_t(\sigma^t(\theta^t))}{\theta_t}, S_t \right) \middle| \theta \right] \end{aligned} \tag{5}$$

I focus on allocations and the marginal distortions, or wedges, they imply for the optimality conditions on labor supply, savings, and consumption decisions. However, tax implementation is beyond the scope of this paper, since it's non-unique and matching household and planner optimality conditions is insufficient to establish implementation. See Golosov et al. (2007) and Kocherlakota (2010) for a further discussion on direct mechanisms and the distinction between their allocations and tax implementation.

The planner's problem is to choose an allocation that maximizes the social welfare function (3) subject to the resource constraint (4), incentive compatibility constraints (5), and pollution accumulation constraints (2). I write $U_{c,t}$, $U_{q,t}$, and $U_{l,t}$ as the marginal utilities of the clean good, dirty good, and labor. Allocations are characterized by labor and savings wedges and by the consumption wedge τ_t^q between the clean and dirty goods.

$$1-\tau_t^y(\theta^t) \equiv -\frac{U_{l,t}}{\theta_t U_{c,t}} \qquad 1-\tau_t^s(\theta^t) \equiv \frac{1}{\beta R}\frac{U_{c,t}}{\mathbb{E}_t[U_{c,t+1}]} \qquad 1+\tau_t^q(\theta^t) \equiv \frac{U_{q,t}}{U_{c,t}}$$

In the next section, I characterize optimal wedges as a function of elasticities and the skill process, first allowing the planner to freely choose any incentive-feasible allocation.

3 Full optimum

3.1 Recursive problem

Global incentive compatibility compares truthful reporting with every possible reporting strategy, so the constraint set is intractable. I follow the first-order approach of Kapička (2013) and Golosov et al. (2016), solving the dual problem of minimizing the present value of resource costs required to deliver a given utility with global constraints replaced by local envelope conditions. Since households are atomistic, a reporting deviation by one does not affect aggregate emissions. Hence, this approach is still valid (see Appendix A.1).

Write $\theta^t = (\theta^{t-1}, \theta)$ and suppose that θ^{t-1} is fixed. Let $u(\theta)$ be the remaining lifetime utility after the current skill draw, where $w(\theta)$ is its continuation value.

$$u(\theta) = U\left(c(\theta), q(\theta), \frac{y(\theta)}{\theta}, S_t\right) + \beta w(\theta) \tag{6}$$

Let $\dot{u}(\theta)$ represent how remaining lifetime utility changes with the current skill draw. Incentive compatibility implies the following local envelope condition, where $w_2(\theta)$ is the marginal continuation value, defined as the derivative of expected continuation utility

with respect to current skill, holding the current report fixed.

$$\dot{u}(\theta) = -U_l\left(c(\theta), q(\theta), \frac{y(\theta)}{\theta}, S_t\right)\frac{y(\theta)}{\theta^2} + \beta w_2(\theta) \tag{7}$$

These equations introduce two continuation promises that tomorrow's allocation must deliver, namely that expected utility must be $w(\theta)$ and the marginal continuation value must be $w_2(\theta)$. The second promise is important because households with different skills can make the same report, receiving the same continuation allocation. However, under persistent shocks, they may face different future skill distributions so that a nearby household contemplating a deviation may end up receiving a different expected utility.

For $t \geq 1$, the planner's current allocation must respect the promise-keeping and threat-keeping constraints inherited from the previous period.

$$\hat{w}_t(\theta^{t-1}) = \int_0^\infty u(\theta) f_t(\theta|\theta_{t-1}) d\theta \tag{8}$$

$$\hat{w}_{2,t}(\theta^{t-1}) = \int_0^\infty u(\theta) f_{2,t}(\theta|\theta_{t-1}) d\theta \tag{9}$$

Let $V_t(\hat{w}_t(\cdot), \hat{w}_{2,t}(\cdot), \mathcal{H}_t)$ be the minimized resource cost of delivering the promised utility and marginal continuation value to the whole population. The two promise functions specify the planner's commitments at every preceding history. Since households share the same pollution stock, continuation costs must be minimized jointly. The planner's cost minimization problem is

$$V_t(\hat{w}_t(\cdot), \hat{w}_{2,t}(\cdot), \mathcal{H}_t) = \min_{c_t, q_t, y_t, u_t, w_t, w_{2,t}, Q_t, S_t} \left\{ \int [c_t + q_t - y_t] dF^t + \frac{1}{R} V_{t+1}(w_t(\cdot), w_{2,t}(\cdot), \mathcal{H}_{t+1}) \right\}$$

The planner's problem is the above subject to the remaining lifetime utility and its derivative (6)-(7), promise-keeping and threat-keeping (8)-(9), aggregate dirty good consumption (1), and pollution accumulation (2). The household constraints hold at each preceding history, whereas emissions and pollution are common to the population.

At date 0, there are no inherited promises. The problem $V_0(\hat{w}_0, \mathcal{H}_0)$ uses the initial skill density f_0 and imposes the following weighted expected utility target.

$$\hat{w}_0 = \int_0^\infty \alpha(\theta) u_0(\theta) dF_0(\theta)$$

The relaxed welfare problem selects the largest $\hat{w}_0$ such that $V_0(\hat{w}_0, \mathcal{H}_0) \leq RK_0$. Write $\bar{\alpha}_0(\theta) = \alpha(\theta)$ and $\bar{\alpha}_t(\theta) = 1$ for $t \geq 1$ to distinguish the initial welfare weights from later promise-keeping constraints. At date T, the continuation promises are fixed at $w_T(\cdot) = w_{2,T}(\cdot) = 0$ and $V_{T+1} = 0$. Since no period after T exists, the planner does not consider subsequent pollution damages.

In the next subsection, I characterize the interior optima of this relaxed problem, assuming continuously differentiable allocation rules and a differentiable continuation value with integrable bounds allowing differentiation under integrals. The density is always positive in the support. The optimality conditions are necessary but not sufficient for global incentive compatibility, which I assess through numerical deviation checks.

3.2 Labor, savings, and consumption wedges

The externality adds no correction term to the labor and savings optimality conditions, though the addition of the second good adds an Edgeworth substitutability term to the labor wedge. Before deriving the wedges implied by the optimal allocation, I define the following elasticities that characterize them.

$$\epsilon_t(\theta) = \frac{U_{ll,t} l_t}{U_{l,t}} \quad \sigma_t(\theta) = -\frac{U_{cc,t} c_t}{U_{c,t}} \quad \sigma_{cq,t}(\theta) = -\frac{U_{cq,t} q_t}{U_{c,t}} \quad \gamma_{c,t}(\theta) = \frac{U_{cl,t} l_t}{U_{c,t}} \quad \gamma_{q,t}(\theta) = \frac{U_{ql,t} l_t}{U_{q,t}}$$

The first two are the inverses of the elasticity of labor supply and of the elasticity of intertemporal substitution. The third is minus the elasticity of the marginal utility of the numéraire with respect to dirty good consumption and is positive when the goods are Edgeworth substitutes. The last two measure complementarity between labor and each of the goods. The strict single crossing condition of Assumption 2.2 implies $1 + \epsilon_t - \gamma_{c,t} > 0$.

Assumption 3.1 (Boundary and integrability)**.**
Let ψ_t and ϕ_t be the multipliers on the local envelope condition (7) and the remaining lifetime utility identity (6), respectively. At each date and last period's history, $\gamma_{c,t}(\theta)/\theta$ is absolutely integrable near zero and

$$\lim_{\theta \downarrow 0} \psi_t(\theta) = \lim_{\theta \to \infty} \psi_t(\theta) = 0$$
$$\lim_{b \to \infty} \exp\left[-\int_0^b \gamma_{c,t}(x) \frac{dx}{x} \right] \psi_t(b) = 0$$

The functions $f_t / U_{c,t}$ and $\bar{\alpha}_t f_t$ are absolutely integrable on $(0, \infty)$ with weight

$$\exp\left[-\int_0^x \gamma_{c,t}(s)\frac{ds}{s}\right]$$

For $t \geq 1$, $f_{2,t}$ is absolutely integrable with this same weight and ϕ_t is absolutely integrable without it. Initial Pareto weights are positive on a set of positive F_0 probability.

Proposition 3.1 (Optimal labor wedge)**.**
Under Assumption 3.1, the labor wedge satisfies the following optimality condition

$$\frac{\tau_t^y}{1-\tau_t^y} = A_t(\theta)B_t(\theta)C_t(\theta) + \beta R \frac{\tau_{t-1}^y}{1-\tau_{t-1}^y} D_t(\theta)$$

where

$$\begin{aligned}
A_t(\theta) &= 1 + \epsilon_t(\theta) - \gamma_{c,t}(\theta) \\
B_t(\theta) &= \frac{1-F_t(\theta)}{\theta f_t(\theta)} \\
C_t(\theta) &= \int_\theta^\infty \exp\left[\int_\theta^x \left(\sigma_t(\tilde{x})\frac{\dot{c}_t(\tilde{x})}{c_t(\tilde{x})} + \sigma_{cq,t}(\tilde{x})\frac{\dot{q}_t(\tilde{x})}{q_t(\tilde{x})} - \gamma_{c,t}(\tilde{x})\frac{\dot{y}_t(\tilde{x})}{y_t(\tilde{x})}\right) d\tilde{x}\right] \\
&\qquad (1-\lambda_{1,t}\bar{\alpha}_t(x)U_{c,t}(x))\,\frac{f_t(x)dx}{1-F_t(\theta)} \\
D_t(\theta) &= \frac{A_t(\theta)}{A_{t-1}}\frac{U_{c,t}(\theta)}{U_{c,t-1}}\frac{\theta_{t-1}\int_\theta^\infty \exp\left[-\int_\theta^x \gamma_{c,t}(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right] f_{2,t}(x)dx}{\theta f_t(\theta)}
\end{aligned}$$

Proof. See Appendix A.3. □

This is the standard labor wedge formula from Golosov et al. (2016), except that C_t now includes the term $\sigma_{cq,t}\dot{q}_t/q_t$. When the goods are Edgeworth substitutes, higher dirty good consumption lowers the marginal utility of the numéraire, holding other arguments fixed. If dirty good consumption increases with skill, this increases the exponent term, affecting the gains from redistribution and insurance.

Proposition 3.2 (Optimal savings wedge)**.**
Let $\widehat{U}_{c,t} \equiv \frac{U_{c,t}}{1-\chi_t}$ and $\chi_t \equiv \frac{\gamma_{c,t}}{1+\epsilon_t-\gamma_{c,t}}\frac{\tau_t^y}{1-\tau_t^y}$. For $t < T$, suppose $1-\chi_t > 0$ at the current history and $1-\chi_{t+1} > 0$ at every next-period history. Savings then satisfy the following

$$\frac{1}{\widehat{U}_{c,t}} = \frac{1}{\beta R}\,\mathbb{E}_t\left[\frac{1}{\widehat{U}_{c,t+1}}\right]$$

Proof. See Appendix A.4. □

This is the generalized inverse Euler condition, expressed as a reweighted inverse Euler condition similar to Hellwig (2021). When preferences are non-separable between labor and consumption, the cost of delivering utility is weighted by $1-\chi_t$. When the numéraire is a labor complement, consumption makes work less painful and shrinks the information rent that truth-telling requires. Neither the second good nor the externality changes the structure of savings wedges.

Proposition 3.3 (Optimal consumption wedge)**.**
The consumption wedge between the clean and dirty goods satisfies

$$\frac{\tau_t^q - \mathrm{SCC}_t}{1+\tau_t^q} = -\frac{\gamma_{q,t}-\gamma_{c,t}}{1+\epsilon_t-\gamma_{c,t}}\frac{\tau_t^y}{1-\tau_t^y}$$

Proof. See Appendix A.5. □

This optimality condition balances the cost of distorting consumption choices on the left-hand side against the benefit of relaxing incentive constraints on the right-hand side, after accounting for pollution. To get an intuition for this condition, consider a utility-compensated reduction in dirty good consumption and its effect on each side.

Each unit of dirty good taken away requires $1+\tau^q$ units of the clean good as compensation. It also saves a unit of this dirty good and avoids emissions, valued at the social cost of carbon. The left-hand side is therefore the marginal cost of the distortion net of the saved dirty good and avoided damages, per unit of the numéraire used as compensation.

If $\tau^y > 0$ and $\gamma_{c,t} \neq \gamma_{q,t}$, meaning there is a motive for redistribution and the consumption mix affects labor disutility, the distortion also has an incentive-provision benefit. When $\gamma_{q,t}-\gamma_{c,t} < 0$, meaning the dirty good is a relative labor substitute, a high-skill household considering misreporting as a low-skill household to be able to work less for the same income ends up valuing the dirty good more than the intended household. When the planner takes away some dirty good consumption and replaces it with more clean good consumption in this low-skill household allocation, the high-skill household may prefer to stay with its intended allocation, relaxing the incentive constraints limiting redistribution and insurance. The planner can use this to increase the earnings it demands from households, with appropriate consumption compensation, whose cost is captured by the labor wedge on the right-hand side. $-(\gamma_{q,t}-\gamma_{c,t})$ measures how much changing the consumption mix discourages imitation, while $1+\epsilon-\gamma_c$ measures how much increasing required earnings encourages it. The ratio reflects how much the planner can ask of households while preserving incentive compatibility. At the optimum, this marginal benefit is equalized with the marginal cost of distorting consumption choices.

Corollary 3.1 (Optimal consumption wedge at the social cost of carbon)**.**

If $\tau_t^y = 0$ or $\gamma_{c,t} = \gamma_{q,t}$, then $\tau_t^q = \text{SCC}_t$.

Proof. Immediate from Proposition 3.3. □

When $\tau^y = 0$ or $\gamma_{c,t} = \gamma_{q,t}$, the consumption wedge is equal to the incentive-adjusted social cost of carbon. Indeed, the right-hand side is then zero and we can rearrange the equation to get $\tau^q = \text{SCC}_t$. There is then no additional incentive correction to the consumption wedge. As I will show in the next subsection, the social cost of carbon can still differ from the standard Pigouvian rule because pollution may affect work incentives.

This is the dynamic analogue to the consumption wedge derived in Jacobs and de Mooij (2015). The intratemporal tradeoff keeps the same form, but the labor wedge term causes the right-hand side of the equation to reflect the incentive benefits on both redistribution across initial types and insurance against subsequent skill shocks. As a consequence, the consumption wedge is not only nonlinear, but also history-dependent. This result extends the dynamic results of Golosov et al. (2003), relaxing the weak separability assumption behind their Atkinson-Stiglitz result.

3.3 Social cost of carbon

Define $\xi_t \equiv \frac{U_{Sl,t} l_t}{U_{S,t}}$ as the complementarity between environmental quality and labor supply. $\xi_t - \gamma_{c,t}$ measures this complementarity relative to that of the numéraire. When it is positive, households working more are willing to give up more clean good consumption for a marginal improvement in environmental quality. When the planner wants to redistribute income and provide insurance against income risk, the social cost of carbon values the damages caused by an additional unit of emissions, adjusted for their effect on incentives.

In the formulas below, $\partial S_{t+j}/\partial Q_t$ is the total effect of current emissions on current and future pollution stocks, including their effects through intermediate pollution stocks. The integrals are taken over the unconditional population distribution F^{t+j} of complete skill histories at each date.

Proposition 3.4 (Social cost of carbon)**.**
The incentive-adjusted social cost of carbon is

$$\text{SCC}_t = -\sum_{j=0}^{T-t} \frac{1}{R^j} \int \left(1 + \frac{\xi_{t+j} - \gamma_{c,t+j}}{1 + \epsilon_{t+j} - \gamma_{c,t+j}} \frac{\tau_{t+j}^y}{1 - \tau_{t+j}^y}\right) \frac{U_{S,t+j}}{U_{c,t+j}} \frac{\partial S_{t+j}}{\partial Q_t} \, dF^{t+j}$$

The social cost of carbon reduces to the standard Pigouvian rule when, at every date and history affected by current emissions, either $\tau_{t+j}^y = 0$ or $\xi_{t+j} = \gamma_{c,t+j}$.

$$\mathrm{SCC}_t^P = -\sum_{j=0}^{T-t} \frac{1}{R^j} \int \frac{U_{S,t+j}}{U_{c,t+j}} \frac{\partial S_{t+j}}{\partial Q_t} \, dF^{t+j}$$

Proof. See Appendix A.6. □

This adjusted social cost of carbon measures the direct effect of pollution on utility, as well as its indirect effect on labor provision incentives. To understand what the bracket term means, suppose the planner considers allowing a marginal increase in the pollution stock, holding utility constant. Since each unit of pollution causes $-U_S/U_c$ in damages in units of the numéraire, the planner must compensate households with $-U_S/U_c$ more of the numéraire to keep them indifferent. Suppose environmental quality is a relative labor complement, so that $\xi - \gamma_c > 0$. Consider a high-skill household contemplating getting the allocation intended for a low-skill household. Because it would work less to produce the same income, it needs less clean consumption to compensate for the environmental harm. Hence, the compensation it gets at its current labor supply is more than it needs to offset its valuation of environmental harm, increasing its incentive to misreport its skill.

This effect can be offset by asking the low-skill household to generate less income and adjusting clean consumption to maintain indifference. Each unit of forgone labor income allows clean consumption to fall by $1 - \tau^y$, leaving a net resource loss of τ^y. Compensating the direct environmental harm therefore leaves an additional resource cost associated with restoring incentive compatibility. $\xi - \gamma_c$ measures how strongly pollution increases deviation incentives, while $1 + \epsilon - \gamma_c$ measures how a reduction in required labor income offsets this increase. The ratio determines how demanded labor income must be adjusted to maintain incentive compatibility. When $\tau^y > 0$, this raises the valuation of pollution above marginal damages.

When environmental quality is instead a relative labor substitute, the argument runs in reverse. This adjusted formula extends that of Jacobs and de Mooij (2015) to a dynamic setting. Current emissions affect incentives not only in the current period but also in every future period when they are persistent. The social cost of carbon therefore depends on the entire life path of the labor wedge, taking into account not only intratemporal but also intertemporal redistribution.

4 Constrained optimum

The results so far let the government reach any incentive-compatible allocation, but actual carbon taxes may be set inside imperfect tax systems where the income tax is not the one a dynamic Mirrlees optimum might prescribe. To model this restriction, I consider

the planner's problem in which labor and savings wedges are held at exogenous schedules. Both this planner and the previous planner maximize the same social welfare function, but its feasible allocation set is limited to the allocations that satisfy the exogenous wedge constraints. If the exogenous wedges are the same as those the planner would want, the restrictions do not bind and the results of the previous section carry over without change. I am not making any judgment about the reasons the planner may be constrained in its choice set, whether this is due to political-economy considerations, lack of coordination between tax authorities, or other factors, and merely taking the constraints for granted.

The planner chooses the best possible allocation under these restrictions, but the model does not specify how this allocation is implemented through taxes and transfers. Hence, this analysis does not speak to the best way to recycle carbon tax revenue. Note however that keeping income wedges fixed implicitly rules out using carbon tax revenue to cut marginal income tax rates, as British Columbia and Sweden did for example. I briefly address this by considering a number of income tax reforms in Section 5, though this is not a comprehensive analysis of the optimal use of carbon tax revenue.

4.1 Fixed income wedges

Let the labor wedge be held at a schedule $\bar{\tau}_t^y(\theta) < 1$ with Lagrange multiplier $\eta_t(\theta) f_t(\theta|\theta_-)$.

$$\Gamma_t(\theta) \equiv -\frac{U_l\left(c(\theta), q(\theta), \frac{y(\theta)}{\theta}, S\right)}{\theta\, U_c\left(c(\theta), q(\theta), \frac{y(\theta)}{\theta}, S\right)} - (1 - \bar{\tau}_t^y(\theta)) = 0 \tag{10}$$

The equality $\Gamma_t(\theta) = 0$ requires the allocation's implied labor wedge to equal the schedule. Moreover, the savings wedge is held at $\bar{\tau}_t^s(\theta_t) < 1$.

$$U_{c,t}(\theta^t) = (1 - \bar{\tau}_t^s(\theta_t))\, \beta R\ \mathbb{E}_t\Big[U_{c,t+1}(\theta^{t+1})\Big] \tag{11}$$

Because the savings-wedge constraint involves current and future consumption, the date-t problem cannot enforce it alone. Given a marginal utility of current clean-good consumption, the constraint requires tomorrow's marginal utility to average some value M, with $\vartheta_{t+1}(\theta^t)$ the multiplier on this promise in next period's problem. Today's consumption is likewise constrained by the requirement $\hat{M}$ inherited from yesterday's problem. This new state is needed because two continuation plans delivering the same promised utility and marginal continuation value can still differ in their expected future marginal utility, even if the savings-wedge constraint only allows one value for the expected future marginal utility. The planner's problem gains the constraints (10) and (11), $M_t(\cdot)$ in V_{t+1}, and state $\hat{M}_t(\cdot)$.

The labor-wedge constraint adds a term to the first order conditions for consumption, labor, and pollution, while the savings-wedge constraint adds two. The first term comes from $\hat{M}$ inherited from the date-$(t-1)$ problem and the second term from M given to the date-$(t+1)$ problem. Combining the savings terms gives

$$\eta_t^s(\theta^t) \equiv \left[\vartheta_t - \frac{\vartheta_{t+1}(\theta^t)}{(1-\bar{\tau}_t^s)\,\beta R^2}\right] U_{c,t}(\theta^t) \tag{12}$$

Hence, η_t prices the labor-wedge constraint and η_t^s combines the contributions of the inherited and future marginal-utility promises. Both enter the first order conditions through $\zeta_t \equiv \eta_t(1-\bar{\tau}_t^y) - \eta_t^s$ and η_t enters once more.

To simplify notation, I define $\mathcal{I}_{c,t}^q = \partial/\partial c_t \ln(U_{q,t}/U_{c,t})$ as the proportional change in the household's marginal willingness to pay for the dirty good, measured in units of the clean good, as it consumes more of the clean good and $\mathcal{I}_{y,t}^q = \partial/\partial y_t \ln(U_{q,t}/U_{c,t})$ as its counterpart for labor income. $\mathcal{I}_{c,t}^S = \partial/\partial c_t \ln(-U_{S,t}/U_{c,t})$ and $\mathcal{I}_{y,t}^S = \partial/\partial y_t \ln(-U_{S,t}/U_{c,t})$ are defined similarly for environmental quality.

Proposition 4.1 (Constrained-optimal consumption wedge)**.**
Let $\tau_t^{y,*}$ be the labor wedge the full optimum would prescribe at the constrained allocation. The constrained-optimal consumption wedge satisfies

$$\frac{\tau_t^q - \mathrm{SCC}_t}{1+\tau_t^q} = -\frac{\gamma_{q,t}-\gamma_{c,t}}{1+\epsilon_t-\gamma_{c,t}}\,\frac{\tau_t^{y,*}}{1-\tau_t^{y,*}} - \eta_t\,\mathcal{I}_{y,t}^q - \zeta_t\,\mathcal{I}_{c,t}^q$$

The social cost of carbon keeps its form, with the adjustment factor replaced by

$$1 + \frac{\xi_{t+j}-\gamma_{c,t+j}}{1+\epsilon_{t+j}-\gamma_{c,t+j}}\,\frac{\tau_{t+j}^{y,*}}{1-\tau_{t+j}^{y,*}} + \eta_{t+j}\,\mathcal{I}_{y,t+j}^S + \zeta_{t+j}\,\mathcal{I}_{c,t+j}^S$$

Proof. See Appendix B.1. □

The first term of the consumption wedge has the same form as that in the full optimum, except the labor wedge is replaced by the desired labor wedge.

The second term reflects the shadow cost of the labor-wedge constraint. A positive η_t means that lowering the labor wedge would reduce the planner's resource cost of delivering an allocation; the planner would like to lower the labor wedge but cannot. For brevity, I will call this "overdistortion". Likewise, I will call the opposite case "underdistortion". $\mathcal{I}_{y,t}^q$ measures how willingness to pay for the dirty good changes with labor income. Suppose

$\eta_t > 0$ and the dirty good is the relative labor substitute, $\mathcal{I}^q_{y,t} < 0$. Then, $-\eta_t \mathcal{I}^q_{y,t} > 0$, pulling up the consumption wedge. Intuitively, a marginal shift from the dirty good to the clean good that leaves utility unchanged reduces the marginal disutility of work.

The third term captures how the consumption mix affects the two wedge constraints through the marginal utility of the clean good. ζ_t is the net shadow price of a proportional change in this marginal utility, combining the labor-wedge and savings-wedge constraint multipliers. $\mathcal{I}^q_{c,t}$ measures how willingness to pay for the dirty good changes with clean-good consumption and is positive when the dirty good is normal. Set the savings multiplier aside and suppose $\eta_t > 0$, so that $\zeta_t = \eta_t(1 - \bar{\tau}^y_t) > 0$. If the dirty good is normal, $\mathcal{I}^q_{c,t} > 0$, then $-\zeta_t \mathcal{I}^q_{c,t} < 0$, pulling down the consumption wedge.

The adjustment factor in the social cost of carbon also gains two terms. If $\eta_{t+j} > 0$ and having a higher labor income raises the willingness to pay for environmental quality, $\mathcal{I}^S_{y,t+j} > 0$, then the term is positive, pulling up the social cost of carbon. If we set aside the savings multiplier η^s_{t+j} in ζ_{t+j} such that $\zeta_{t+j} > 0$ and if consuming more of the clean good raises the willingness to pay for environmental quality, $\mathcal{I}^S_{c,t+j} > 0$, then the term is once again positive, further pulling up the social cost of carbon. Intuitively, a marginal improvement in environmental quality offset by lower clean consumption so that utility remains unchanged reduces the marginal disutility of work and raises the marginal utility of the clean good. Both channels strengthen marginal work incentives and raise the social value of environmental quality when labor is overdistorted.

Corollary 4.1 (Constrained-optimal consumption wedge with separable preferences)**.** Under weak separability or $\gamma_{c,t} = \gamma_{q,t}$,

$$\frac{\tau^q_t - \mathrm{SCC}_t}{1 + \tau^q_t} = -\zeta_t \mathcal{I}^q_{c,t}$$

When the dirty good is normal, the consumption wedge is below (above) the social cost of carbon where $\zeta_t > 0$ ($\zeta_t < 0$).

Proof. Proposition 4.1 at $\gamma_{c,t} = \gamma_{q,t}$. □

If labor or savings wedges are fixed, then weak separability no longer ensures that the consumption wedge is set at the social cost of carbon. When consumption moves the willingness to pay for the dirty good, the consumption wedge can be used to offset the wedge constraints.

4.1.1 Savings-wedge constraint over the lifecycle

Whereas the sign of the labor-wedge multiplier is determined by whether labor is underdistorted or overdistorted at a given date and history, acting as a local "push", that of the savings-wedge multiplier pulls in opposite directions at different dates, acting as a "pivot" across the lifecycle.

Lemma 4.1 (Sign of the savings multiplier)**.**
For any labor-wedge schedule and skill process, the savings multiplier takes both signs along every history on which it is not identically zero. If $T \geq 1$ and

$$\mathbb{E}_t\left[\left|\eta^s_{t+1}\frac{\sigma_{t+1}}{c_{t+1}}\right|\right] < \infty \qquad\qquad \eta^s_t\frac{\sigma_t}{c_t} > (1-\bar{\tau}^s_t)\,\mathbb{E}_t\left[\eta^s_{t+1}\frac{\sigma_{t+1}}{c_{t+1}}\right] \tag{13}$$

hold almost surely at every $t < T$, then $\eta^s_0 > 0$ and $\eta^s_T < 0$ almost surely. Reversing the inequality in (13) at every $t < T$ reverses the endpoint signs.

Proof. See Appendix B.2. □

Intuitively, consider the deterministic case where savings is always overdistorted. Because savings has a lower return, the household has an incentive to consume more today and less tomorrow. The planner would like to counteract this and can use the consumption mix across dates to do so. When the dirty good is normal, the planner can partly counteract this frontloading of consumption by pulling the consumption wedge up early in life and down later in life, making future consumption relatively cheaper.

Under uncertainty, risk can reinforce or offset the effect of the savings constraint on the direction of the pivot, depending on whether delivering additional utility is more or less costly in future states where the marginal utility of clean good consumption is high. When condition (13) holds and the dirty good is normal at the initial and terminal dates, the savings correction pulls the consumption wedge up early in life and down later in life.

4.2 Age-dependent and linear consumption wedges

Aside from income wedge constraints, the planner may be unable to set allocations that imply different consumption wedges across households. In my analysis, a linear carbon tax has a rate constant across households and ages. An age-dependent carbon tax has a rate common across households at each age but allowed to vary across ages. Suppose first that income wedges are now free to adjust, but the allocation must be such that the consumption wedge must be common across all households at a given date. Let $s_{qq,t} < 0$

denote the compensated response of dirty good demand to its price holding labor and pollution fixed.

$$\varpi_t \equiv \frac{\bar{\tau}_t^q - \mathrm{SCC}_t}{1 + \bar{\tau}_t^q} \qquad\qquad \varpi_t^* \equiv -\left(\gamma_{q,t} - \gamma_{c,t}\right) \frac{\psi U_{c,t}}{\theta f_t}$$

Proposition 4.2 (Optimal age-dependent consumption wedge)**.**
With income wedges free to adjust, the age-dependent consumption wedge satisfies

$$\varpi_t = \frac{\mathbb{E}_0[|s_{qq,t}|\ \varpi_t^*]}{\mathbb{E}_0[|s_{qq,t}|]}$$

Proof. See Appendices C.2 and C.4. □

The common correction ϖ_t is a weighted average of the desired corrections to the social cost of carbon, with greater weight on households whose dirty-good demand responds more to a compensated price change. When the desired correction exceeds the common correction, raising the price contributes to reducing the planner's resource cost. When it falls short of this correction, this pulls the price down. The optimum balances this across households. This is similar to the multiple-agent Ramsey rule of Diamond (1975). As shown in Appendix C.4, this restriction can also change the social cost of carbon. A similar rule applies when income wedges are fixed.

Corollary 4.2 (Optimal age-dependent consumption wedge with fixed income wedges)**.**
Define

$$\varpi_t^{**} \equiv \varpi_t^* - \left(\gamma_{q,t} - \gamma_{c,t}\right) \frac{\eta_t}{y_t} - \zeta_t \mathcal{I}_{c,t}^q$$

With labor and savings wedges fixed, the age-dependent consumption wedge satisfies

$$\varpi_t = \frac{\mathbb{E}_0[|s_{qq,t}|\ \varpi_t^{**}]}{\mathbb{E}_0[|s_{qq,t}|]}$$

Proof. See Appendices C.3 and C.4. □

Under weak separability, $\varpi_t^{**} = -\zeta_t \mathcal{I}_{c,t}^q$, so the common wedge equals the social cost of carbon when $\mathbb{E}_0[|s_{qq,t}|\zeta_t \mathcal{I}_{c,t}^q] = 0$. A wedge held constant over the lifecycle must balance marginal resource costs across households as well as dates.

In the next section, I use Proposition 4.2 and Corollary 4.2 to quantify the welfare effects

of constraining the consumption wedge to be uniform across households at each age (age-dependent carbon tax) or constant across households and ages (linear carbon tax), using a model calibrated to US data.

5 Quantitative exercise

The theory suggests that a carbon tax should be nonlinear, but what are the likely welfare gains of implementing more flexible carbon pricing instruments? When income taxes are fixed, is using a nonlinear or even age-dependent carbon tax a good substitute for income tax reform when the latter is difficult to implement? To answer these questions, I solve the model calibrated to US data, with an energy composite good standing in for the dirty good and other types of consumption standing in for the clean good.

I follow the quantitative setup and solution method of Farhi and Werning (2013). I extend their logarithmic consumption preferences to two goods, using a CES aggregate with a weight on the energy good that depends on labor supply.

$$U(c,q,l) = \log\left(\left[(1-b_q)c^{\frac{\varsigma-1}{\varsigma}} + b_q e^{-\varkappa l^\alpha} q^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}\right) - \frac{l^\alpha}{\alpha}$$

b_q is the weight of energy in the utility function, calibrated to match the observed share of energy in total consumption, about 7% in the 2023 Consumer Expenditure Survey (U.S. Bureau of Labor Statistics 2024). ς is the elasticity of substitution between the two goods, set to 0.4 so that the implied own-price elasticity is about -0.4. West and Williams (2007) report that gasoline is a relative leisure complement (labor substitute), so I set $\varkappa$ to be positive, at 0.25. Households work from age 25 to 64 and are retired from 65 to 84. Since retirees do not work, there is no more screening problem after age 64.

Since a single cohort in a single country cannot generate a realistic path for the social cost of carbon, I hold the social cost of carbon fixed at a value consistent with \$190/tCO_2, the central value for a ton emitted in 2020 according to the EPA (U.S. Environmental Protection Agency 2023). This means that any deviation from the social cost of carbon is solely due to redistribution and not due to the evolution of climate damages over time. Because the social cost of carbon in the consumption wedge is common across all households, it affects the level but not the shape of the optimal carbon tax. Hence, this simplification does not affect the qualitative distributional implications. The results can be interpreted as a cross-section of a cohort in a given year in an overlapping generations model with no intergenerational transfers where the carbon tax schedule is fixed over one's lifetime.

I compare carbon pricing when income wedges are optimized and when they are fixed. I calibrate the labor wedge using the Heathcote–Storesletten–Violante (HSV) formula (Heathcote et al. 2017; Heathcote and Tsujiyama 2021), with level parameter $\lambda^T = 0.840$ and progressivity $p = 0.181$. Since there is no HSV-style calibration for the capital income tax, I set a constant savings wedge of 0.9%, equivalent to the average effective tax rate of 18% reported by the CBO[4].

Table 1: Calibration

Parameter or variable	Value	Calibration basis
Discount factor, β	0.95	$\beta R = 1$, from Farhi and Werning (2013)
Labor-disutility curvature, α	3	Farhi and Werning (2013)
Log-skill innovation variance	0.0095	Farhi and Werning (2013)
Energy expenditure share	7%	U.S. Bureau of Labor Statistics (2024)
Substitution elasticity, ς	0.4	Labandeira et al. (2017)
Leisure complementarity, $\varkappa$	0.25	Sign motivated by West and Williams (2007)
Labor progressivity, p	0.181	Heathcote et al. (2017)
Labor schedule level, λ^T	0.840	Heathcote and Tsujiyama (2021)
Savings wedge, $\bar{\tau}^s$	0.009	Congressional Budget Office (2014)

Under my assumptions, as shown in Figure 1, the average optimal labor wedge rises more steeply with age than the US-calibrated labor wedge and the planner's preferred savings wedge is decreasing in age. Since it is plausible that the US-calibrated wedges can be rationalized with a different calibration on preferences and the skill shock process, I am not making a statement about the optimality of the US tax system. Therefore, I also consider a second exercise where income wedges are fixed at schedules that would have been optimal in a world without externalities. This simulates the case of a government that had already optimized its tax system, but cannot readjust it after introducing carbon taxes. I then compare the welfare gains from using more flexible carbon tax instruments against those of an income tax reform, measured as departures from US-calibrated income wedges towards the planner's preferred wedges.

[4]Congressional Budget Office 2014

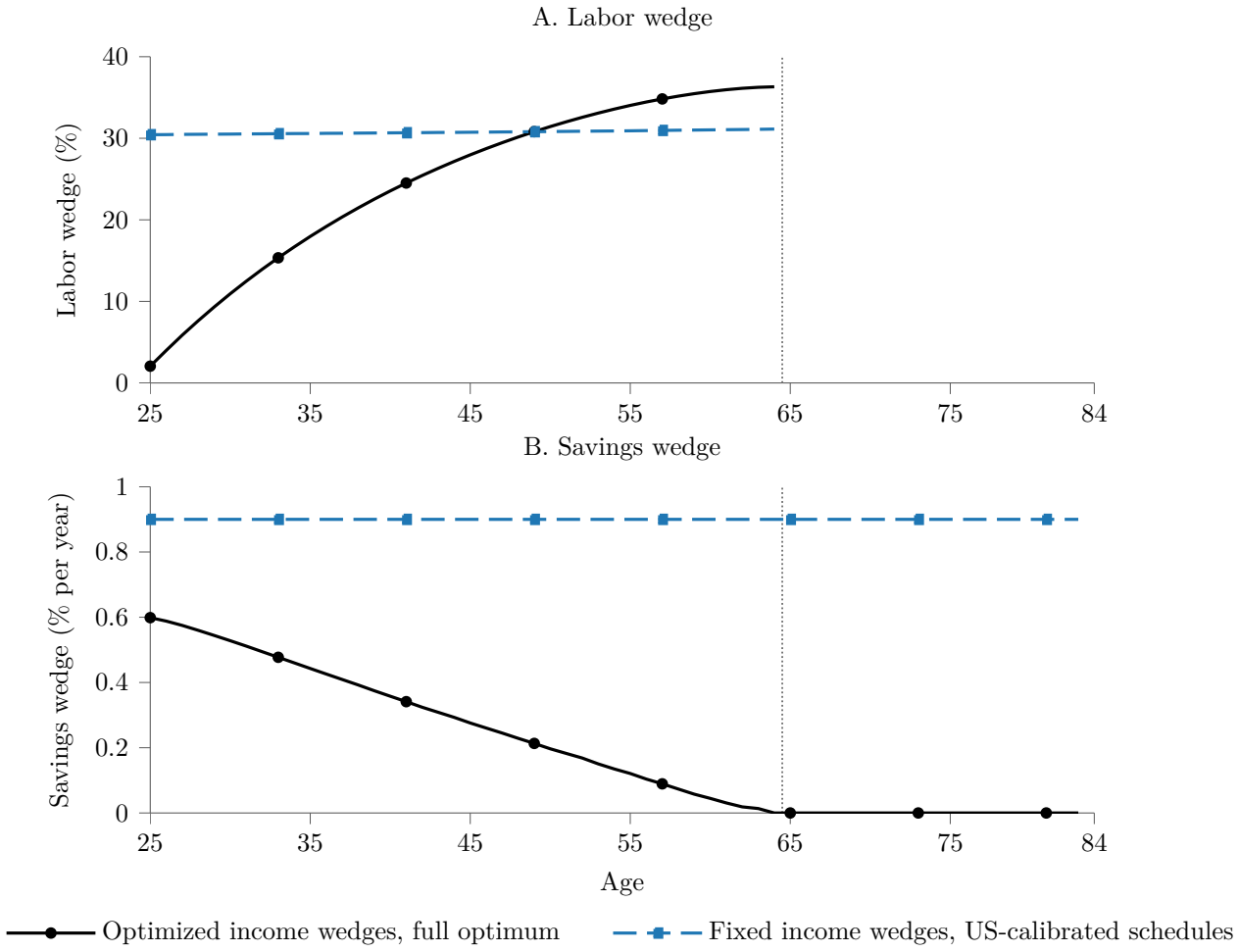


Figure 1: Average labor and savings wedges by age, with income wedges optimized or fixed.

5.1 Progressivity of fully nonlinear carbon taxes

What does an age-dependent or fully nonlinear carbon tax look like and how progressive is it? Figure 2 plots the average optimal consumption wedge over the lifecycle, comparing age-dependent and fully nonlinear consumption wedges in a world with optimized labor and savings wedges and in a world with fixed labor and savings wedges.

In a world with optimized labor and savings wedges, the average optimal fully nonlinear carbon tax starts slightly above the social cost of carbon of \$190/tCO_2 and rises steadily to about \$243/tCO_2 by the end of the working life. Afterwards, it drops to the social cost of carbon, since households enter retirement and there is no longer a screening problem. The best age-dependent carbon tax tracks the average optimal fully nonlinear carbon tax almost perfectly and is therefore not plotted in the figure. Hence, with optimized income wedges, incentive-provision calls for a carbon tax that is higher than the social cost of carbon during the working life.

In a world with fixed labor and savings wedges, the average fully nonlinear carbon tax starts much higher than the social cost of carbon, at \$285/tCO$_2$, and declines steadily until crossing the social cost of carbon at 37 and then becoming a subsidy from age 64. This pattern mostly comes from the fixed savings wedge. Since the savings wedge is too high at every age, households consume too much early in life, and a carbon tax that falls with age makes later consumption relatively cheaper, which works like a subsidy to saving. The age-dependent carbon tax follows a similar pattern.

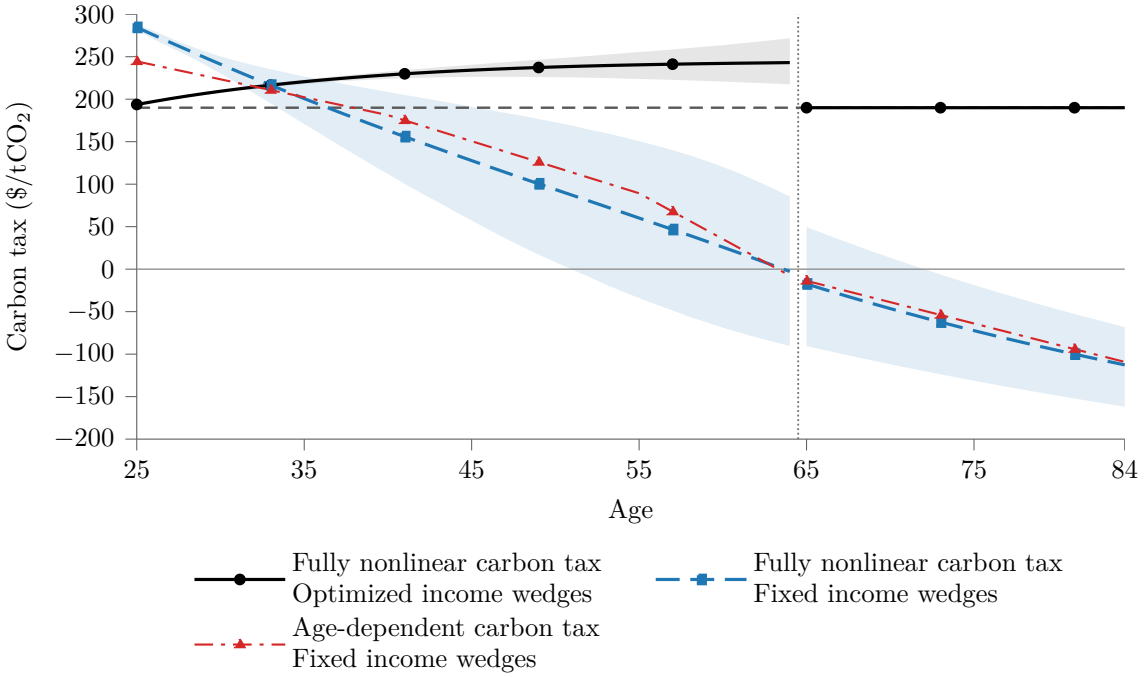


Figure 2: Average carbon tax by age, with cross-household 10th-90th percentile bands. Gray dashes mark the social cost of carbon (\$190/tCO$_2$), dots mark retirement at 65.

Figure 3 compares average marginal carbon tax rates across quintiles of current labor income at three ages. At age 35, the gap between the highest and lowest quintiles is only \$2.42/tCO$_2$ when labor and savings wedges are optimized. With income wedges fixed at US-calibrated levels, the rate rises from \$174.49/tCO$_2$ in the lowest quintile to \$222.00/tCO$_2$ in the highest, a gap of \$47.51/tCO$_2$. By age 50, the gap reaches \$158.61/tCO$_2$ with fixed wedges and \$23.10/tCO$_2$ with optimized wedges.

At age 64, the average marginal tax exceeds the social cost of carbon of \$190/tCO$_2$ in every quintile when income wedges are optimized. It rises from \$217.35/tCO$_2$ in the lowest labor income quintile to \$272.11/tCO$_2$ in the highest. With fixed income wedges, the lowest quintile on average receives a subsidy at age 52. By age 64, the bottom three quintiles on average face a subsidy, and the rate rises from −\$87.97/tCO$_2$ in the lowest quintile to \$87.12/tCO$_2$ in the highest. The gap of \$175.09/tCO$_2$ is a bit over three times the gap of \$54.76/tCO$_2$ under optimized income wedges.

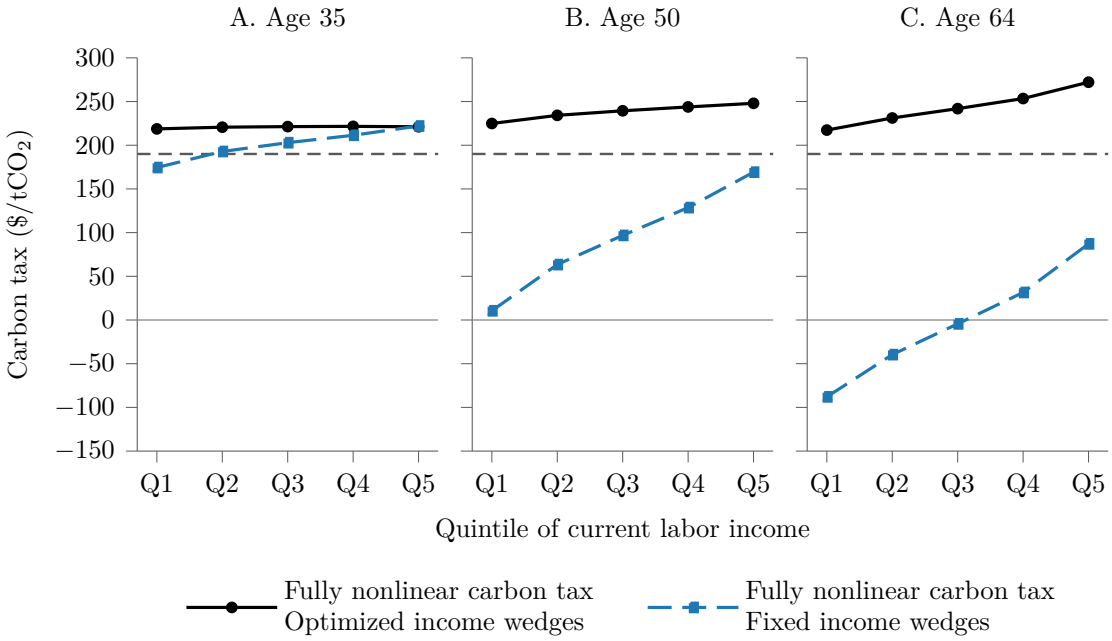


Figure 3: Average marginal nonlinear carbon tax at ages 35, 50 and 64. Households are ranked by current labor income within each age and income-tax regime. Gray dashes mark the social cost of carbon, \$190/tCO$_2$.

These results suggest a significant redistributive role for carbon taxation, but this might be due to the US-calibrated income wedges being far from what the planner would like. Is there still a significant role for redistribution when the carbon tax is introduced to a world with optimized income taxes, but the latter are simply not allowed to readjust afterwards?

5.2 Consumption wedges alongside previously optimal income wedges

Suppose the government initially optimizes its income tax system without accounting for climate damages or using consumption taxes. It then introduces a carbon tax. However, labor and savings wedges must remain fixed at their baseline values at each history, while consumption, earnings and implied transfers can adjust. Does this lack of coordination call for a change in the way the carbon tax is set?

Table 2 compares the gains from more flexible carbon instruments when labor and savings wedges are fixed at levels that would have been optimal in a world without externalities or consumption taxation.

Table 2: Welfare gains with fixed income wedges

Carbon tax instrument	With climate damages	Without climate damages
Linear carbon tax	0.00000	0.00000
Age-dependent carbon tax	0.00037	0.00031
Fully nonlinear carbon tax	0.00741	0.00599

Notes: Consumption-equivalent welfare gains, in % of lifetime consumption, relative to the best linear consumption wedge in each scenario. Labor and savings wedges are fixed at their values from the optimum without climate damages or consumption taxation.

The best linear rate becomes about \$206/tCO$_2$ when income wedges can adjust, adding only 0.000059% of consumption. At the social cost of carbon, the gain from reoptimization of income wedges is below 0.000001%. With income wedges fixed, age-dependent consumption wedges range from about \$197 to \$219/tCO$_2$ (Figure 4). The age-dependent wedge rises during working life and falls close to the social cost of carbon at retirement. Average fully nonlinear consumption wedges follow a similar path. The linear wedge, about \$205/tCO$_2$, captures 97.6% of the gain from introducing the optimal fully nonlinear consumption wedge. Age differentiation captures 4.9% of the additional gain beyond the linear wedge.

Having a non-zero consumption wedge between the two goods is already useful without environmental damages. Without externalities, age differentiation adds 0.000309% relative to the best linear rate. A fully nonlinear wedge adds a welfare gain equivalent to 0.005991% of lifetime consumption.

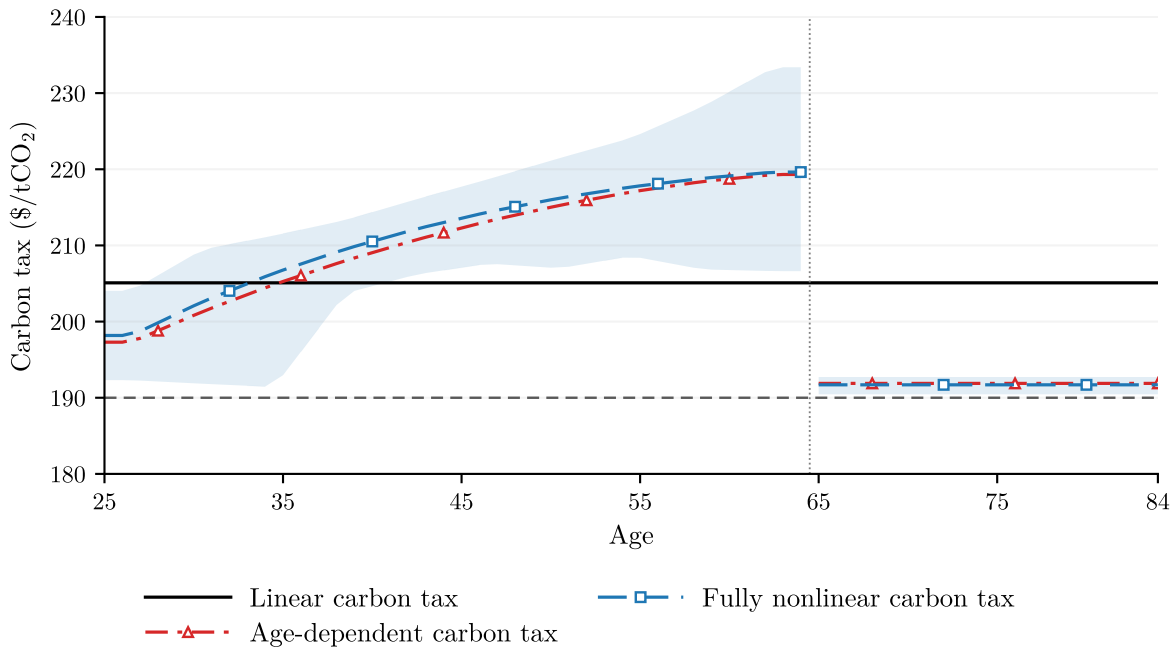


Figure 4: Average optimal carbon taxes with labor and savings wedges fixed at levels that would have been optimal in a world without externalities or consumption taxation. Shading shows the 10th-90th percentiles of the fully nonlinear carbon tax. Gray dashes mark the social cost of carbon ($190/tCO$_2$), dots mark retirement at 65.

Hence, only having access to linear carbon taxes entails little welfare cost since the previously optimized income wedges are already close to what the planner would like. However, if income wedges are not initially optimal, is allowing for a more flexible carbon tax an adequate substitute for income tax reform?

5.3 Income tax reform or carbon tax reform?

I return to the US-calibrated income wedges, which differ from the planner's preferred wedges. What is the welfare gain from allowing the consumption wedge to be age-dependent, or even fully nonlinear? How does this compare with the welfare gain from going from the US-calibrated income wedges to the planner's preferred wedges?

Table 3 reports the welfare gains relative to US-calibrated income wedges without a carbon tax. With optimized income wedges, a linear consumption wedge delivers 21.5% more welfare gain. However, making the consumption wedge age-dependent or fully nonlinear has little value, as the linear consumption wedge case already captures 99% of the gain from the fully nonlinear consumption wedge. This is because distorting consumption choices has little role in redistribution when income wedges are optimal.

On the other hand, with income wedges fixed at US-calibrated schedules, flexibility in the

consumption wedge is more important. The consumption-equivalent welfare gain is 50% larger with a fully nonlinear consumption wedge than with a linear one. The age-dependent consumption wedge captures about 87% of this additional gain, so age-dependence alone contributes most of the gains of a fully nonlinear consumption wedge.

Table 3: Welfare gains from carbon and income tax reform

	Carbon tax instrument			
Income wedges	None	Linear	Age-dependent	Fully nonlinear
US-calibrated	0.0000	0.1877	0.2691	0.2818
Reoptimized	1.4569	1.7706	1.7720	1.7723

Notes: Consumption-equivalent welfare gains, in % of lifetime consumption, relative to US-calibrated income schedules without a carbon tax. Linear consumption wedges are common across households and ages. Age-dependent consumption wedges are common within each age. Fully nonlinear consumption wedges can also differ across households. Labor and savings wedges are fixed in the US-calibrated row and jointly reoptimized in the second row.

However, the gains from more flexible consumption wedges are dwarfed by those of reoptimizing income wedges. Even holding consumption wedges linear, going from the US-calibrated income wedges to the planner's preferred wedges adds 1.5793% of consumption, over 16 times the gain from making the consumption wedge fully nonlinear. Given the large differences between the US-calibrated and planner's preferred income wedges, it might be informative to compare more modest reforms. I consider a decrease of 1 percentage point in the capital income tax rate, decreases of 1-2 percentage points in labor income tax rates, increases in the HSV progressivity parameter, capital income tax cuts in retirement since the model calls for a zero savings wedge after retirement, and graduated labor income tax cuts for younger workers since this is where the labor wedge is furthest from the planner's preferred labor wedge. Because age-dependent carbon taxation captures most of the additional welfare gain of a fully nonlinear carbon tax, I compare these gains against it. Table 4 reports the welfare gains of these limited reforms and the size of the reforms that would match the welfare gains of age-dependent carbon taxation.

Table 4: Welfare gains from limited income tax reforms

Instrument	Reform	Welfare gain	Equivalent reform
Capital income tax	18% → 17%	0.2807	18% → 17.13%
Labor income tax	−1 p.p.	0.2553	−1.22 p.p.
	−2 p.p.	0.3164	
HSV progressivity (p)	0.181 → 0.231	0.2204	0.181 →≈ 0.313
	0.181 → 0.310	0.2670	
	0.181 → 0.350	0.2927	
Retiree capital income tax	18% → 9%	0.2626	18% →≈ 8.08%
	18% → 0%	0.3249	
Graduated youth relief	From −2 p.p. at 25 to −0.5 p.p. at 32–34	0.2734	–

Notes: Welfare gains are in % of lifetime consumption, relative to US-calibrated income schedules without a carbon tax. All income reforms retain a linear consumption wedge of 0.40 (about 153.85/tCO_2). Equivalent reforms match the 0.2691% gain from age-dependent carbon taxation. Retirement relief leaves working-life savings wedges unchanged. The youth profile cuts labor income rates by 2, 1.5, 1 and 0.5 p.p. at ages 25, 26-28, 29-31 and 32-34.

The first experiment is the equivalent of a broad capital income tax cut of 1 percentage point, which delivers a welfare gain of 0.2807%, exceeding the gains of age-dependent carbon taxation. In the model, this is the equivalent of reducing the savings wedge by 0.05 percentage points. The second experiment is the equivalent of a broad labor income tax cut of 1 or 2 percentage points, which delivers a welfare gain of 0.2553% or 0.3164%, respectively. A cut of about 1.22 percentage points would match the welfare gain of implementing the best age-dependent carbon tax. The third experiment considers different increases in the HSV progressivity parameter, by 0.05, by 0.129, and by 0.169. The welfare gain is smaller. Matching the gain from age-dependent carbon taxation requires increasing the parameter by about 0.132, or 72.9%. The fourth experiment cuts retiree capital income taxes in half and then also considers eliminating them, with a roughly 55.1% cut being sufficient to match the welfare gains of an age-dependent carbon tax. The fifth experiment implements a graduated youth relief, with cuts of 2, 1.5, 1 and 0.5 percentage points at ages 25, 26-28, 29-31 and 32-34. This particular ladder is illustrative, but is meant to take into account that the difference between US-calibrated and planner's preferred labor wedges is largest at younger ages. The resulting welfare gain is about the same as that of the age-dependent carbon tax.

These examples suggest that small broad-based labor and capital tax cuts can deliver enough welfare to match the gains of an age-dependent carbon tax. If the tax system is particularly suboptimal for some ages, a targeted reform can work too. However, making the labor income tax more progressive requires an implausibly large increase in the progressivity parameter.

In short, even in a world where the income tax system implies wedges that are far from where the planner would want them to be, the welfare gain from implementing an age-dependent carbon tax can be matched by modest cuts in labor or capital income tax rates. Moreover, non-targeted income tax cuts can still be consistent with the objectives of a redistributive planner when income taxes are overall higher than where they need to be. In the US-calibrated example, labor wedges are furthest above the planner's preferred levels for younger workers, and savings wedges are always too high. The difference between observed and preferred wedges depends on the country and the problem's assumptions, but the takeaway remains that carbon pricing flexibility is a poor substitute for income tax reform.

6 Conclusion

In this paper, I explore the optimal design of carbon taxes when the government's goal is to correct for a climate externality while also redistributing income in a world where households face labor income shocks. In a dynamic Mirrlees model with two goods, I analytically show that whether households should face different carbon taxes depends on how consumption choices interact with labor supply decisions. When the dirty good is a relative labor complement, the optimal carbon tax is lower than the social cost of carbon. When labor and savings wedges are fixed at potentially suboptimal levels, the optimal carbon tax may have further correction terms offsetting these constraints.

The quantitative exercise shows that the consumption wedge does not need to be significantly adjusted even when income wedges were set without accounting for climate externalities and cannot be reoptimized, and that the welfare gains from making the consumption wedge age-dependent or fully nonlinear are small relative to modest income tax reforms. Although the theory calls for nonlinear history-dependent carbon taxes, the numerical results therefore suggest that the redistributive role of carbon taxation is limited.

Hence, at least within the confines of a Mirrlees-type analysis, there is little practical benefit to deviating from the standard Pigouvian rule of setting the carbon tax equal to the social cost of carbon, with redistributive goals best achieved with limited income tax reform. If marginal income taxes cannot change, an age-dependent carbon tax can capture most of the gains of a fully nonlinear carbon tax.

This paper has a number of limitations that would be interesting to address in future work. First, it focuses on allocations rather than tax implementations, so it cannot speak to the specifics of how an age-dependent or even fully nonlinear carbon tax could be best put in place, or to the best way to recycle revenue from a carbon tax. Second, the model features

a single cohort, so a natural extension is considering an overlapping generations model to study intergenerational redistribution. This would also allow for the implementation of the framework in a full integrated assessment model to better study the implications of income inequality for emissions.

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A Optimal wedge derivations

A.1 First-order approach with pollution

Under Assumptions 2.1–2.2 and the regularity conditions in Section 3.1, consider a globally incentive-compatible allocation and its induced pollution path. A household has zero mass, so changing its reports leaves aggregate emissions unchanged at every date. Pollution accumulation then implies that the entire pollution path is unchanged.

I fix a past skill history and, for ease of notation, omit the argument. For $t < T$, let $u_{t+1}(r, x)$ be future utility from truthful reporting after current report r and next skill x. Because skills are Markov and the pollution path is fixed, this utility depends on the true past history only through x, even if r was a misreport. Households with true current skill θ that report r and report truthfully thereafter receive

$$J_t(r, \theta) = U\left(c_t(r), q_t(r), \frac{y_t(r)}{\theta}, S_t\right) + \beta \int_0^\infty u_{t+1}(r, x) f_{t+1}(x|\theta) dx$$

The transition density depends on true skill, while the continuation allocation depends on the report. Global incentive compatibility makes $r = \theta$ optimal at almost every history θ^t. Continuity and positive densities extend this condition to interior histories in the support. For any fixed report r, $u_t(\theta) - J_t(r, \theta)$ is non-negative and equals zero at $\theta = r$. Its derivative with respect to true skill is therefore zero at $\theta = r$, giving

$$\dot{u}_t(\theta) = -U_{l,t} \frac{y_t(\theta)}{\theta^2} + \beta w_{2,t}(\theta) \qquad w_{2,t}(\theta) = \int_0^\infty u_{t+1}(\theta, x) f_{2,t+1}(x|\theta) dx$$

The derivative defining $w_{2,t}$ holds the reported continuation allocation fixed. At $t = T$, the continuation terms are zero. Taking expected remaining utility and differentiating it with respect to past skill, holding the allocation fixed, gives (8) and (9).

Every globally incentive-compatible allocation satisfying regularity conditions is therefore feasible for the relaxation. If a global optimum of the relaxed problem also satisfies global incentive compatibility, no allocation in this class that is feasible for the original welfare problem can yield higher welfare.

A.2 First order conditions

Because $U(c, q, l, S)$ can be written as $U(c, q, y/\theta, S)$, I write U_θ as $-U_l l/\theta$. To derive the first order conditions of the planner's problem, I keep the continuation value at the population level. At each preceding history θ^{t-1}, write $f = f_t(\theta|\theta_{t-1})$ and $f_2 = \frac{\partial f_t}{\partial \theta_{t-1}}$. The

multipliers $\lambda_{1,t}$ and $\lambda_{2,t}$ depend on the preceding history, while ϕ_t and ψ_t also depend on current skill. I normalize these multipliers relative to the probability measure dF^{t-1} and suppress history and time arguments where there is no ambiguity.

Let ν_t be the multiplier on aggregate emissions and let μ_t be the shadow cost of the current pollution stock, including its effects on future pollution stocks. The Lagrangian is

$$\begin{aligned}\mathcal{L}_t = \int \Bigg\{ \int_0^\infty \Bigg(& [c + (1+\nu_t)q - \theta l] f \\ & + \psi \left[-U_l \frac{l}{\theta} + \beta w_2 - \dot{u} \right] - \lambda_1 u \bar{\alpha}_t f + \lambda_2 u f_2 \\ & + \phi[u - U - \beta w] \Bigg) d\theta + \lambda_1 \hat{w}_t - \lambda_2 \hat{w}_{2,t} \Bigg\} dF^{t-1} \\ & + \frac{1}{R} V_{t+1}(w_t(\cdot), w_{2,t}(\cdot), \mathcal{H}_{t+1}) \\ & - \nu_t Q_t - \mu_t [S_t - \mathcal{S}(\mathcal{H}_t, Q_t)]\end{aligned}$$

For a variation δu in remaining lifetime utility, integrating the $-\psi\, \delta\dot{u}$ term by parts gives

$$\delta \mathcal{L}_t = \int \left\{ -[\psi\, \delta u]_0^\infty + \int_0^\infty [\dot{\psi} + \phi - \lambda_1 \bar{\alpha}_t f + \lambda_2 f_2] \delta u \, d\theta \right\} dF^{t-1}$$

The first order conditions are:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial c} &: f - \psi U_{cl} \frac{l}{\theta} = \phi U_c \\ \frac{\partial \mathcal{L}_t}{\partial q} &: (1+\nu_t) f - \psi U_{ql} \frac{l}{\theta} = \phi U_q \\ \frac{\partial \mathcal{L}_t}{\partial l} &: -\theta f - \phi U_l = \frac{1}{\theta} \psi \left[U_{ll} l + U_l \right] \\ \frac{\partial \mathcal{L}_t}{\partial u} &: \dot{\psi} = \lambda_1 \bar{\alpha}_t f - \lambda_2 f_2 - \phi\end{aligned}$$

For $t < T$, consider variations δw_t and $\delta w_{2,t}$ in the continuation promises. Holding $\mathcal{H}_{t+1}$ fixed, the envelope theorem applied to next period's problem gives

$$\delta V_{t+1} = \int [\lambda_{1,t+1} \delta w_t - \lambda_{2,t+1} \delta w_{2,t}] dF^t$$

Since $dF^t = f_t \, d\theta \, dF^{t-1}$, the corresponding variation of the Lagrangian is

$$\begin{aligned}\delta\mathcal{L}_t &= \int\int_0^\infty [-\beta\phi_t\delta w_t + \beta\psi_t\delta w_{2,t}]d\theta\, dF^{t-1} + \frac{1}{R}\delta V_{t+1} \\ &= \int\left[\left(\frac{\lambda_{1,t+1}}{R} - \frac{\beta\phi_t}{f_t}\right)\delta w_t + \left(\frac{\beta\psi_t}{f_t} - \frac{\lambda_{2,t+1}}{R}\right)\delta w_{2,t}\right] dF^t\end{aligned}$$

At the optimum, the coefficients of these independent variations are zero when $f_t > 0$, so

$$\lambda_{1,t+1}(\theta^t) = \beta R\frac{\phi_t(\theta^t)}{f_t(\theta_t|\theta_{t-1})} \qquad \lambda_{2,t+1}(\theta^t) = \beta R\frac{\psi_t(\theta^t)}{f_t(\theta_t|\theta_{t-1})} \tag{14}$$

Pollution affects all households. If we differentiate the Lagrangian with respect to S_t,

$$\mu_t = -\int\left[\frac{\psi_t}{f_t}U_{Sl,t}\frac{l_t}{\theta_t} + \frac{\phi_t}{f_t}U_{S,t}\right] dF^t + \frac{1}{R}\frac{\partial V_{t+1}}{\partial S_t} \tag{15}$$

Let $S_q \equiv \frac{\partial \mathcal{S}}{\partial Q_t}$ be the marginal effect of current emissions on the current pollution stock. Holding the promise functions and S_t fixed, differentiating with respect to Q_t gives

$$\frac{\partial\mathcal{L}_t}{\partial Q_t} = -\nu_t + \mu_t S_q + \frac{1}{R}\frac{\partial V_{t+1}}{\partial Q_t} = 0 \qquad \Longrightarrow \qquad \nu_t = \mu_t S_q + \frac{1}{R}\frac{\partial V_{t+1}}{\partial Q_t} \tag{16}$$

The three wedges can be derived from here.

A.3 Labor wedge

Dividing the first order condition for c of Appendix A.2 by U_c and using $\gamma_c \equiv U_{cl}l/U_c$,

$$\phi = \frac{f}{U_c} - \frac{\psi}{\theta}\gamma_c \tag{17}$$

From the first order condition for l and since $U_{ll}l + U_l = U_l(1+\epsilon)$ with $\epsilon \equiv U_{ll}l/U_l$,

$$-\theta f - \phi U_l = \frac{\psi}{\theta}U_l(1+\epsilon) \implies \phi = -\frac{\theta f}{U_l} - \frac{\psi}{\theta}(1+\epsilon)$$

Equating the two expressions for ϕ and collecting the terms in ψ,

$$\frac{f}{U_c} + \frac{\theta f}{U_l} = -\frac{\psi}{\theta}(1+\epsilon-\gamma_c)$$

The labor wedge is $1-\tau^y = -U_l/(\theta U_c)$, so $\theta U_c/U_l = -1/(1-\tau^y)$, making the left-hand side $\frac{f}{U_c}\left(1-\frac{1}{1-\tau^y}\right) = -\frac{f}{U_c}\frac{\tau^y}{1-\tau^y}$. Substituting and rearranging,

$$\frac{\tau^y}{1-\tau^y} = (1+\epsilon-\gamma_c)\frac{\psi U_c}{\theta f} \tag{18}$$

Next, substitute ϕ from (17) into $\dot{\psi} = \lambda_1\bar{\alpha}_t f - \lambda_2 f_2 - \phi$,

$$\dot{\psi} - \frac{\gamma_c}{\theta}\psi = \lambda_1\bar{\alpha}_t f - \lambda_2 f_2 - \frac{f}{U_c}$$

This is a linear first-order ordinary differential equation. I use the integrating factor $\exp\left[-\int_0^\theta \gamma_c(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right]$. Integrating from θ to $b > \theta$ and dividing by the factor at θ leaves the upper boundary term $\exp\left[-\int_\theta^b \gamma_c(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right]\psi(b)$. For any fixed $\theta > 0$, the weighted boundary condition in Assumption 3.1 makes this term disappear as $b \to \infty$, giving

$$\psi(\theta) = \int_\theta^\infty \exp\left[-\int_\theta^x \gamma_c(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right]\left(\frac{f(x)}{U_c(x)} - \lambda_1\bar{\alpha}_t(x)f(x) + \lambda_2 f_2(x)\right)dx \tag{19}$$

Under Assumption 3.1, absolute integrability near zero makes the integrating factor converge to one as θ approaches 0. Absolute convergence of the weighted integrals then allows taking this limit in (19). Using $\lim_{\theta\to 0}\psi(\theta) = 0$ and rearranging gives

$$\lambda_{1,t} = \frac{\int_0^\infty \exp\left[-\int_0^x \gamma_{c,t}(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right]\left(\frac{1}{U_{c,t}(x)}f_t(x) + \lambda_{2,t}f_{2,t}(x)\right)dx}{\int_0^\infty \exp\left[-\int_0^x \gamma_{c,t}(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right]\bar{\alpha}_t(x)f_t(x)dx}$$

The denominator is finite and positive by Assumption 3.1. Substituting (19) into (18),

$$\frac{\tau^y}{1-\tau^y} = (1+\epsilon-\gamma_c)\frac{U_c(\theta)}{\theta f(\theta)}\int_\theta^\infty \exp\left[-\int_\theta^x \gamma_c(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right]\left(\frac{f(x)}{U_c(x)} - \lambda_1\bar{\alpha}_t(x)f(x) + \lambda_2 f_2(x)\right)dx$$

Next, I rewrite the first part of the integral using

$$\frac{d}{d\theta}\ln U_{c,t} = \frac{U_{cc}\dot{c} + U_{cq}\dot{q} + U_{cl}\dot{l}}{U_c} = -\sigma_t\frac{\dot{c}}{c} - \sigma_{cq,t}\frac{\dot{q}}{q} + \gamma_c\frac{\dot{l}}{l}$$

Since $l = y/\theta$ gives $\dot{l}/l = \dot{y}/y - 1/\theta$, integrating from θ to x and multiplying by the integrating factor gives the following:

$$\frac{U_{c,t}(\theta)}{U_{c,t}(x)} \exp\left[-\int_\theta^x \gamma_c(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right] = \exp\left[\int_\theta^x \left(\sigma_t(\tilde{x})\frac{\dot{c}_t(\tilde{x})}{c_t(\tilde{x})} + \sigma_{cq,t}(\tilde{x})\frac{\dot{q}_t(\tilde{x})}{q_t(\tilde{x})} - \gamma_c(\tilde{x})\frac{\dot{y}_t(\tilde{x})}{y_t(\tilde{x})}\right) d\tilde{x}\right]$$

The first part of the labor wedge is therefore

$$(1+\epsilon_t(\theta)-\gamma_{c,t}(\theta))\frac{1}{\theta f_t(\theta)}\int_\theta^\infty \exp\left[\int_\theta^x \left(\sigma_t(\tilde{x})\frac{\dot{c}_t(\tilde{x})}{c_t(\tilde{x})} + \sigma_{cq,t}(\tilde{x})\frac{\dot{q}_t(\tilde{x})}{q_t(\tilde{x})} - \gamma_{c,t}(\tilde{x})\frac{\dot{y}_t(\tilde{x})}{y_t(\tilde{x})}\right) d\tilde{x}\right] \\ (1-\lambda_{1,t}\bar{\alpha}_t(x)U_{c,t}(x))\, f_t(x)\, dx$$

For the second part of the labor wedge, use (14) and shift the date back one period,

$$\lambda_{2,t}(\theta^{t-1}) = \beta R \frac{\psi_{t-1}}{f_{t-1}}$$

Next, evaluating (18) at $t-1$,

$$\frac{\psi_{t-1}}{f_{t-1}} = \frac{\tau^y_{t-1}}{1-\tau^y_{t-1}} \frac{\theta_{t-1}}{U_{c,t-1}\left(1+\epsilon_{t-1}-\gamma_{c,t-1}\right)}$$

The $\lambda_2 f_2$ term of the wedge is $\lambda_{2,t}$, which does not depend on current skill, times the same terms before the term in parentheses.

$$(1+\epsilon_t(\theta)-\gamma_{c,t}(\theta))\frac{U_{c,t}(\theta)}{\theta f_t(\theta)}\lambda_{2,t}\int_\theta^\infty \exp\left[-\int_\theta^x \gamma_{c,t}(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right] f_{2,t}(x)\, dx$$

Substituting the two and rearranging,

$$\beta R \frac{\tau^y_{t-1}}{1-\tau^y_{t-1}} \frac{1+\epsilon_t(\theta)-\gamma_{c,t}(\theta)}{1+\epsilon_{t-1}-\gamma_{c,t-1}} \frac{U_{c,t}(\theta)}{U_{c,t-1}} \frac{\theta_{t-1}\int_\theta^\infty \exp\left[-\int_\theta^x \gamma_{c,t}(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right] f_{2,t}(x)\, dx}{\theta f_t(\theta)}$$

Putting together the two and multiplying and dividing the first term by $1-F_t(\theta)$,

$$\frac{\tau^y_t}{1-\tau^y_t} = A_t(\theta)B_t(\theta)C_t(\theta) + \beta R \frac{\tau^y_{t-1}}{1-\tau^y_{t-1}} D_t(\theta)$$

with

$$A_t(\theta) = 1 + \epsilon_t(\theta) - \gamma_{c,t}(\theta)$$

$$B_t(\theta) = \frac{1 - F_t(\theta)}{\theta f_t(\theta)}$$

$$C_t(\theta) = \int_\theta^\infty \exp\left[\int_\theta^x \left(\sigma_t(\tilde{x})\frac{\dot{c}_t(\tilde{x})}{c_t(\tilde{x})} + \sigma_{cq,t}(\tilde{x})\frac{\dot{q}_t(\tilde{x})}{q_t(\tilde{x})} - \gamma_{c,t}(\tilde{x})\frac{\dot{y}_t(\tilde{x})}{y_t(\tilde{x})}\right) d\tilde{x}\right] (1 - \lambda_{1,t}\bar{\alpha}_t(x)U_{c,t}(x)) \frac{f_t(x)dx}{1 - F_t(\theta)}$$

$$D_t(\theta) = \frac{A_t(\theta)}{A_{t-1}} \frac{U_{c,t}(\theta)}{U_{c,t-1}} \frac{\theta_{t-1} \int_\theta^\infty \exp\left[-\int_\theta^x \gamma_{c,t}(\tilde{x})\frac{d\tilde{x}}{\tilde{x}}\right] f_{2,t}(x)dx}{\theta f_t(\theta)}$$

A.4 Savings wedge

The savings wedge follows from the same first order conditions of Appendix A.2. I define χ_t, a term reducing the cost of delivering utility when consumption complements labor.

$$\chi_t(\theta^t) \equiv \frac{\gamma_{c,t}}{1 + \epsilon_t - \gamma_{c,t}} \frac{\tau_t^y}{1 - \tau_t^y}$$

Using $\frac{\tau_t^y}{1-\tau_t^y} = \frac{\psi U_c}{\theta f}(1 + \epsilon_t - \gamma_{c,t})$, $\phi = f/U_c - \psi\gamma_c/\theta$ from (17) can be rewritten as follows:

$$\frac{\phi}{f} = \frac{1}{U_c}\left(1 - \gamma_c \frac{\psi U_c}{\theta f}\right) = \frac{1 - \chi_t}{U_{c,t}} \tag{20}$$

$\chi_t > 0$ when the numéraire is complementary with labor and labor is distorted and zero when preferences are separable between the numéraire and labor or when labor is undistorted. The utility-promise condition in (14) gives

$$\lambda_{1,t+1}(\theta^t) = \beta R \frac{\phi(\theta)}{f(\theta)} = \beta R \frac{1 - \chi_t}{U_{c,t}} \tag{21}$$

For $t \geq 1$, using the endpoint and integrability conditions in Assumption 3.1,

$$0 = \int_0^\infty \dot{\psi}\, d\theta = \lambda_{1,t} \int_0^\infty \bar{\alpha}_t f\, d\theta - \lambda_{2,t} \int_0^\infty f_2\, d\theta - \int_0^\infty \phi\, d\theta$$

Since $\bar{\alpha}_t = 1$ and $\int_0^\infty f_2\, d\theta = 0$, $\lambda_{1,t} = \int_0^\infty \phi\, d\theta$, which by (20) is

$$\lambda_{1,t} = \mathbb{E}_{t-1}\left[\frac{1 - \chi_t}{U_{c,t}}\right] \tag{22}$$

For $t < T$, evaluating (22) at $t + 1$ and equating with (21) gives:

$$\frac{1-\chi_t}{U_{c,t}} = \frac{1}{\beta R}\,\mathbb{E}_t\left[\frac{1-\chi_{t+1}}{U_{c,t+1}}\right]$$

When $1-\chi_t > 0$ at the current history and $1-\chi_{t+1} > 0$ at every next-period history, define the incentive-adjusted marginal utility $\widehat{U}_{c,t} \equiv U_{c,t}/(1-\chi_t)$. The inverse Euler condition becomes

$$\frac{1}{\widehat{U}_{c,t}} = \frac{1}{\beta R}\,\mathbb{E}_t\left[\frac{1}{\widehat{U}_{c,t+1}}\right]$$

A.5 Consumption wedge

Starting from the first order conditions of Appendix A.2,

$$\phi = \frac{f}{U_c} - \frac{\psi U_{cl}\frac{l}{\theta}}{U_c}$$

Because c and q share the same ϕ,

$$\phi = \frac{f}{U_q} - \frac{\psi U_{ql}\frac{l}{\theta}}{U_q} + \frac{\nu_t f}{U_q}$$

Hence,

$$\frac{f}{U_q} - \frac{\psi U_{ql}\frac{l}{\theta}}{U_q} + \frac{\nu_t f}{U_q} = \frac{f}{U_c} - \frac{\psi U_{cl}\frac{l}{\theta}}{U_c}$$

Multiplying by U_q and U_c,

$$U_c\left(f - \psi U_{ql}\frac{l}{\theta} + \nu_t f\right) = U_q\left(f - \psi U_{cl}\frac{l}{\theta}\right)$$

Using $\gamma_c \equiv \frac{U_{cl}\,l}{U_c}$ and $\gamma_q \equiv \frac{U_{ql}\,l}{U_q}$ or, equivalently, $\gamma_c U_c = U_{cl}\,l$ and $\gamma_q U_q = U_{ql}\,l$,

$$U_c\left(f - \frac{\psi}{\theta}\gamma_q U_q + \nu_t f\right) = U_q\left(f - \frac{\psi}{\theta}\gamma_c U_c\right)$$

Then, putting the γ terms (as well as the externality term) on one side,

$$U_c f - U_q f = \frac{\psi}{\theta}U_c U_q(\gamma_q - \gamma_c) - \nu_t U_c f$$

Finally, dividing by $U_q f$,

$$\frac{U_c}{U_q} - 1 = \frac{\psi U_c}{\theta f}(\gamma_q - \gamma_c) - \nu_t\frac{U_c}{U_q}$$

Note that, by definition, $1+\tau^q = U_q/U_c$, so $\frac{U_c}{U_q} - 1 = -\frac{\tau^q}{1+\tau^q}$. Multiplying through by -1,

$$\frac{\tau^q}{1+\tau^q} = -\frac{\psi U_c}{\theta f}(\gamma_q - \gamma_c) + \nu_t \frac{U_c}{U_q}$$

Since $\frac{U_c}{U_q} = \frac{1}{1+\tau^q}$,

$$\frac{\tau^q}{1+\tau^q} = -\frac{\psi U_c}{\theta f}(\gamma_q - \gamma_c) + \frac{\nu_t}{1+\tau^q}$$

Rearranging,

$$\frac{\tau^q - \nu_t}{1+\tau^q} = -\frac{\psi U_c}{\theta f}(\gamma_q - \gamma_c)$$

Note that, from the labor wedge derivation,

$$\frac{\psi U_c}{\theta f} = \frac{1}{1+\epsilon_t - \gamma_{c,t}} \frac{\tau_t^y}{1-\tau_t^y}$$

Substituting,

$$\frac{\tau_t^q - \nu_t}{1+\tau_t^q} = -\frac{\gamma_{q,t} - \gamma_{c,t}}{1+\epsilon_t - \gamma_{c,t}} \frac{\tau_t^y}{1-\tau_t^y}$$

A.6 Social cost of carbon

Substituting $\phi = f/U_c - \psi U_{cl} l/(\theta U_c)$ from the clean good condition into (15) and moving the continuation term to the left-hand side gives

$$\mu_t - \frac{1}{R}\frac{\partial V_{t+1}}{\partial S_t} = \int \left(-\frac{\psi}{\theta f} U_{Sl} l - \left[1 - \frac{\psi}{\theta f} U_{cl} l\right] \frac{U_S}{U_c} \right) dF^t$$

Rewrite $\frac{\psi}{\theta f}$ using (18),

$$\frac{\psi}{\theta f} = \left(\frac{\tau^y}{1-\tau^y}\right) \frac{1}{U_c} \frac{1}{1+\epsilon-\gamma_c}$$

Hence, also defining $\xi \equiv \frac{U_{Sl} l}{U_S}$,

$$\begin{aligned}
\mu_t - \frac{1}{R}\frac{\partial V_{t+1}}{\partial S_t} &= \int \left(-\left(\frac{\tau^y}{1-\tau^y}\right) \frac{1}{1+\epsilon-\gamma_c} \frac{U_{Sl} l}{U_c} - \left[1 - \left(\frac{\tau^y}{1-\tau^y}\right) \frac{1}{1+\epsilon-\gamma_c} \frac{U_{cl} l}{U_c}\right] \frac{U_S}{U_c} \right) dF^t \\
&= \int \left(-\left(\frac{\tau^y}{1-\tau^y}\right) \frac{1}{1+\epsilon-\gamma_c} \frac{U_{Sl} l}{U_c} \frac{U_S}{U_S} - \left[1 - \left(\frac{\tau^y}{1-\tau^y}\right) \frac{\gamma_c}{1+\epsilon-\gamma_c}\right] \frac{U_S}{U_c} \right) dF^t \\
&= \int \left(-\left(\frac{\tau^y}{1-\tau^y}\right) \frac{\xi}{1+\epsilon-\gamma_c} \frac{U_S}{U_c} - \left[1 - \left(\frac{\tau^y}{1-\tau^y}\right) \frac{\gamma_c}{1+\epsilon-\gamma_c}\right] \frac{U_S}{U_c} \right) dF^t \\
&= -\int \left[1 + \left(\frac{\tau^y}{1-\tau^y}\right) \frac{\xi-\gamma_c}{1+\epsilon-\gamma_c}\right] \frac{U_S}{U_c} dF^t
\end{aligned}$$

Each integral runs over the date-t population of complete skill histories. The household quantities and multipliers depend on these histories while μ_t is common to the population.

Both the current pollution stock S_t and current emissions Q_t can affect future pollution. Write $\mathcal{S}_j = \mathcal{S}(\mathcal{H}_j, Q_j)$ for pollution stock accumulation at date j. By the envelope theorem, the effect of past pollution stocks or emissions on the continuation value is the discounted sum of their direct effects on later accumulation constraints. For $X_t \in \{S_t, Q_t\}$,

$$\frac{\partial V_{t+1}}{\partial X_t} = \sum_{j=t+1}^{T} \frac{1}{R^{j-t-1}} \mu_j \frac{\partial \mathcal{S}_j}{\partial X_t}$$

Each derivative of $\mathcal{S}_j$ holds its other arguments fixed. Substituting into (16) gives

$$\nu_t = \sum_{j=t}^{T} \frac{1}{R^{j-t}} \mu_j \frac{\partial \mathcal{S}_j}{\partial Q_t} \tag{23}$$

At the terminal date, $V_{T+1} = 0$, so μ_T equals the current-date integral. At earlier dates, μ_t also includes the future costs caused by the persistence of the current pollution stock. By the chain rule,

$$\frac{\partial S_j}{\partial Q_t} = \frac{\partial \mathcal{S}_j}{\partial Q_t} + \sum_{k=t}^{j-1} \frac{\partial \mathcal{S}_j}{\partial S_k} \frac{\partial S_k}{\partial Q_t} \qquad j \geq t \tag{24}$$

The sum is empty at $j = t$, so $\partial S_t / \partial Q_t = S_q$. Subtracting the continuation term from each pollution stock shadow cost, multiplying by $\frac{1}{R^{j-t}} \partial S_j / \partial Q_t$, and summing over j,

$$\begin{aligned}\sum_{j=t}^{T}\frac{1}{R^{j-t}}\left(\mu_j-\frac{1}{R}\frac{\partial V_{j+1}}{\partial S_j}\right)\frac{\partial S_j}{\partial Q_t} &= \sum_{j=t}^{T}\frac{\mu_j}{R^{j-t}}\left[\frac{\partial S_j}{\partial Q_t}-\sum_{k=t}^{j-1}\frac{\partial \mathcal{S}_j}{\partial S_k}\frac{\partial S_k}{\partial Q_t}\right]\\ &= \sum_{j=t}^{T}\frac{\mu_j}{R^{j-t}}\frac{\partial \mathcal{S}_j}{\partial Q_t}\\ &= \nu_t\end{aligned}$$

The first equality uses the continuation envelope condition and reorders the finite sums. The second uses (24) and the last follows from (23). Substituting the current-date expression above into the left-hand side gives

$$\mathrm{SCC}_t \equiv \nu_t = -\sum_{j=0}^{T-t}\frac{1}{R^j}\int\left(1+\frac{\xi_{t+j}-\gamma_{c,t+j}}{1+\epsilon_{t+j}-\gamma_{c,t+j}}\frac{\tau^y_{t+j}}{1-\tau^y_{t+j}}\right)\frac{U_{S,t+j}}{U_{c,t+j}}\frac{\partial S_{t+j}}{\partial Q_t}\,dF^{t+j}$$

B Derivations with fixed income wedges

B.1 Constrained-optimal consumption wedge

The labor-wedge constraint involves only the current allocation and pollution stock, so it adds a pointwise multiplier but no new state. For savings, define the following

$$M_t(\theta^t) \equiv \frac{U_{c,t}(\theta^t)}{(1-\bar{\tau}_t^s(\theta_t))\beta R} \qquad t = 0, \ldots, T-1$$

The savings-wedge constraint requires $\mathbb{E}_t[U_{c,t+1}] = M_t(\theta^t)$. For $t \geq 1$, last period's problem brings $\hat{M}_t(\theta^{t-1})$, which enters as a new state.

$$\hat{M}_t(\theta^{t-1}) = \int U_c \left(c(\theta), q(\theta), \frac{y(\theta)}{\theta}, S \right) f_t(\theta \mid \theta_-)\, d\theta$$

Let $\vartheta_t(\theta^{t-1})$ be the multiplier on this inherited promise. The inherited-promise term is absent at $t = 0$ and the continuation terms are absent at $t = T$, so $\vartheta_0 = \vartheta_{T+1} = 0$. The constrained Lagrangian $\mathcal{L}_t$ is the population Lagrangian in Appendix A.2, with the continuation term replaced by

$$\frac{1}{R} V_{t+1}\left(w_t(\cdot), w_{2,t}(\cdot), M_t(\cdot), \mathcal{H}_{t+1}\right)$$

The Lagrangian also acquires the following terms.

$$\int \left\{ \int_0^\infty \eta_t \Gamma_t f_t \, d\theta + \vartheta_t \left[\int_0^\infty U_{c,t} f_t \, d\theta - \hat{M}_t \right] \right\} dF^{t-1}$$

Holding $\mathcal{H}_{t+1}$ fixed, the envelope theorem applied to next period's problem gives

$$\delta V_{t+1} = \int \left[\lambda_{1,t+1} \delta w_t - \lambda_{2,t+1} \delta w_{2,t} - \vartheta_{t+1} \delta M_t \right] dF^t \qquad t < T$$

For a variation in $z \in \{c, q, l, S\}$, the definition of M_t gives

$$\delta M_t = \frac{U_{cz,t}}{(1-\bar{\tau}_t^s)\beta R} \delta z$$

Using $dF^t = f_t \, d\theta \, dF^{t-1}$ and discounting continuation costs by $1/R$, the inherited promise and the continuation value's dependence on M_t contribute

$$\left[\vartheta_t - \frac{\vartheta_{t+1}}{(1-\bar{\tau}_t^s)\beta R^2}\right] U_{cz} f = \frac{\eta_t^s}{U_c} U_{cz} f$$

From here,

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial c}: &\quad f - \psi U_{cl}\frac{l}{\theta} + \eta f \Gamma_c + \frac{\eta_t^s}{U_c} U_{cc} f = \phi U_c \\
\frac{\partial \mathcal{L}_t}{\partial q}: &\quad (1+\nu_t) f - \psi U_{ql}\frac{l}{\theta} + \eta f \Gamma_q + \frac{\eta_t^s}{U_c} U_{cq} f = \phi U_q \\
\frac{\partial \mathcal{L}_t}{\partial l}: &\quad -\theta f - \phi U_l + \eta f \Gamma_l + \frac{\eta_t^s}{U_c} U_{cl} f = \frac{\psi}{\theta}(U_{ll} l + U_l) \\
\frac{\partial \mathcal{L}_t}{\partial u}: &\quad \dot{\psi} = \lambda_1 \bar{\alpha}_t f - \lambda_2 f_2 - \phi \\
\frac{\partial \mathcal{L}_t}{\partial w}: &\quad \lambda_{1,t+1} = \beta R \frac{\phi}{f} \\
\frac{\partial \mathcal{L}_t}{\partial w_2}: &\quad \lambda_{2,t+1} = \beta R \frac{\psi}{f} \\
\frac{\partial \mathcal{L}_t}{\partial S}: &\quad \mathbb{E}_0\left[-\frac{\psi}{f} U_{Sl}\frac{l}{\theta} - \frac{\phi}{f} U_S + \eta \Gamma_S + \frac{\eta_t^s}{U_c} U_{cS}\right] = \mu_t - \frac{1}{R}\frac{\partial V_{t+1}}{\partial S_t}
\end{aligned}$$

Here, $\nu_t = \mu_t S_q + \frac{1}{R}\frac{\partial V_{t+1}}{\partial Q_t}$. The pollution condition averages component contributions over all date-t histories. The continuation derivatives hold the promise functions $w_t(\cdot)$, $w_{2,t}(\cdot)$, and $M_t(\cdot)$, and other inherited pollution arguments fixed. The effect through M_t is already included in η_t^s. At the constrained allocation, $\Gamma_z = -U_{lz}/(\theta U_c) - (1-\bar{\tau}_t^y)U_{cz}/U_c$. Defining $\xi_{c,t} \equiv c_t U_{cS,t}/U_{S,t}$,

$$\begin{aligned}
\Gamma_c &= (1-\bar{\tau}_t^y)\frac{\sigma}{c} - \frac{\gamma_c}{y} \\
\Gamma_q &= (1-\bar{\tau}_t^y)\frac{\sigma_{cq}}{q} - \frac{\gamma_q(1+\tau^q)}{y} \\
\Gamma_l &= (1-\bar{\tau}_t^y)\frac{\epsilon - \gamma_c}{l} \\
\Gamma_S &= -\frac{U_S}{U_c}\left[\frac{\xi}{y} + (1-\bar{\tau}_t^y)\frac{\xi_c}{c}\right]
\end{aligned}$$

Eliminating ϕ between the first order conditions for the clean and dirty goods gives

$$\frac{\tau_t^q - \nu_t}{1+\tau_t^q} = -(\gamma_{q,t} - \gamma_{c,t})\frac{\psi U_{c,t}}{\theta f_t} - \eta_t \Gamma_{c,t} + \eta_t^s \frac{\sigma_t}{c_t} + \frac{\eta_t \Gamma_{q,t} - \eta_t^s \frac{\sigma_{cq,t}}{q_t}}{1+\tau_t^q}$$

Putting the η_t and η_t^s terms together,

$$\frac{\tau_t^q - \nu_t}{1+\tau_t^q} = -(\gamma_{q,t} - \gamma_{c,t})\frac{\psi U_{c,t}}{\theta f_t} - \eta_t \left[\Gamma_{c,t} - \frac{\Gamma_{q,t}}{1+\tau_t^q}\right] + \eta_t^s \left[\frac{\sigma_t}{c_t} - \frac{\sigma_{cq,t}}{q_t(1+\tau_t^q)}\right]$$

Using the definitions for Γ_c and Γ_q,

$$\Gamma_{c,t} - \frac{\Gamma_{q,t}}{1+\tau_t^q} = (1-\bar{\tau}_t^y)\left[\frac{\sigma_t}{c_t} - \frac{\sigma_{cq,t}}{q_t(1+\tau_t^q)}\right] + \frac{\gamma_{q,t} - \gamma_{c,t}}{y_t}$$

The bracketed expression also appears in the η_t^s term. Combining these terms,

$$\frac{\tau_t^q - \nu_t}{1+\tau_t^q} = -(\gamma_{q,t} - \gamma_{c,t})\frac{\psi U_{c,t}}{\theta f_t} - \eta_t \frac{\gamma_{q,t} - \gamma_{c,t}}{y_t} - ((1-\bar{\tau}_t^y)\eta_t - \eta_t^s)\left[\frac{\sigma_t}{c_t} - \frac{\sigma_{cq,t}}{q_t(1+\tau_t^q)}\right]$$

Define $\mathcal{I}_{c,t}^q = \frac{\partial}{\partial c_t}\ln\left(\frac{U_{q,t}}{U_{c,t}}\right)$ and $\mathcal{I}_{y,t}^q = \frac{\partial}{\partial y_t}\ln\left(\frac{U_{q,t}}{U_{c,t}}\right)$, representing how the willingness to pay for the dirty good responds to changes in clean good consumption and in labor income, respectively. Note that, using the elasticity definitions from the main text,

$$\begin{aligned}
\frac{\partial}{\partial c_t}\ln\left(\frac{U_{q,t}}{U_{c,t}}\right) &= \frac{U_{cq,t}}{U_{q,t}} - \frac{U_{cc,t}}{U_{c,t}} = \frac{\sigma_t}{c_t} - \frac{\sigma_{cq,t}}{q_t}\frac{U_{c,t}}{U_{q,t}} = \frac{\sigma_t}{c_t} - \frac{\sigma_{cq,t}}{q_t(1+\tau_t^q)} \\
\frac{\partial}{\partial y_t}\ln\left(\frac{U_{q,t}}{U_{c,t}}\right) &= \frac{U_{qy,t}}{U_{q,t}} - \frac{U_{cy,t}}{U_{c,t}} = \frac{1}{\theta}\left(\frac{U_{ql,t}}{U_{q,t}} - \frac{U_{cl,t}}{U_{c,t}}\right) = \frac{\gamma_{q,t} - \gamma_{c,t}}{y_t}
\end{aligned}$$

Defining $\zeta_t = (1-\bar{\tau}_t^y)\eta_t - \eta_t^s$ and using the above equations,

$$\frac{\tau_t^q - \nu_t}{1+\tau_t^q} = -(\gamma_{q,t} - \gamma_{c,t})\frac{\psi U_{c,t}}{\theta f_t} - \eta_t \mathcal{I}_{y,t}^q - \zeta_t \mathcal{I}_{c,t}^q$$

If we define $\tau_t^{y,*}$ as the desired labor wedge, we can also write

$$\frac{\tau_t^q - \nu_t}{1+\tau_t^q} = -\frac{\gamma_{q,t} - \gamma_{c,t}}{1+\epsilon_t - \gamma_{c,t}}\frac{\tau_t^{y,*}}{1-\tau_t^{y,*}} - \eta_t \mathcal{I}_{y,t}^q - \zeta_t \mathcal{I}_{c,t}^q$$

It remains to calculate the current-date component of the stock cost. Using $U_{Sl}l = \xi U_S$ and $U_{cS}/U_c = \xi_c(U_S/U_c)/c$,

$$-\frac{U_S}{U_c}\left[1+(\xi-\gamma_c)\frac{\psi U_c}{\theta f}+\eta\frac{\xi-\gamma_c}{y}+((1-\bar{\tau}_t^y)\eta-\eta_t^s)\,\frac{\sigma+\xi_c}{c}\right]$$

Define $\mathcal{I}_{c,t}^S=\frac{\partial}{\partial c_t}\ln\left(-\frac{U_{S,t}}{U_{c,t}}\right)$ and $\mathcal{I}_{y,t}^S=\frac{\partial}{\partial y_t}\ln\left(-\frac{U_{S,t}}{U_{c,t}}\right)$, representing how changes in clean good consumption and labor income affect the willingness to pay for environmental quality. Using the elasticity definitions,

$$\begin{aligned}\frac{\partial}{\partial c_t}\ln\left(-\frac{U_{S,t}}{U_{c,t}}\right)&=\frac{U_{cS,t}}{U_{S,t}}-\frac{U_{cc,t}}{U_{c,t}}=\frac{\xi_{c,t}+\sigma_t}{c_t}\\ \frac{\partial}{\partial y_t}\ln\left(-\frac{U_{S,t}}{U_{c,t}}\right)&=\frac{1}{\theta_t}\left(\frac{U_{Sl,t}}{U_{S,t}}-\frac{U_{cl,t}}{U_{c,t}}\right)=\frac{\xi_t-\gamma_{c,t}}{y_t}\end{aligned}$$

From here,

$$\begin{aligned}\mu_t-\frac{1}{R}\frac{\partial V_{t+1}}{\partial S_t}&=-\mathbb{E}_0\left[\left(1+(\xi-\gamma_c)\frac{\psi U_c}{\theta f}+\eta\mathcal{I}_y^S+\zeta_t\mathcal{I}_c^S\right)\frac{U_S}{U_c}\right]\\&=-\mathbb{E}_0\left[\left(1+\frac{\xi-\gamma_c}{1+\epsilon-\gamma_c}\frac{\tau_t^{y,*}}{1-\tau_t^{y,*}}+\eta\mathcal{I}_y^S+\zeta_t\mathcal{I}_c^S\right)\frac{U_S}{U_c}\right]\end{aligned}\tag{25}$$

B.2 Savings multiplier over the lifecycle

Fix a realized history. Let $\kappa_t=(1-\bar{\tau}_t^s)\beta R^2>0$, $P_t=\prod_{j<t}\kappa_j$ with $P_0=1$, and $x_t=\vartheta_t/P_t$. Equation (12) gives

$$x_{t+1}=x_t-\frac{\eta_t^s}{P_tU_{c,t}}\qquad t<T\tag{26}$$

The endpoint conditions $\vartheta_0=\vartheta_{T+1}=0$ imply $x_0=0$ and $x_T=\eta_T^s/(P_TU_{c,T})$. Summing,

$$\sum_{t=0}^{T}\frac{\eta_t^s}{P_tU_{c,t}}=0\tag{27}$$

If any multiplier along the history is nonzero, terms of both signs must appear. Let $\widetilde{\chi}_t$ denote the net adjustment to the cost of delivering utility in the constrained optimum, combining the screening correction with the effects of the wedge restrictions.

$$\begin{aligned}\widetilde{\chi}_t &\equiv \gamma_{c,t}\frac{\psi U_{c,t}}{\theta f_t} - \eta_t \Gamma_{c,t} + \eta_t^s \frac{\sigma_t}{c_t} \\ &= \frac{\gamma_{c,t}}{1+\epsilon_t-\gamma_{c,t}}\frac{\tau_t^{y,*}}{1-\tau_t^{y,*}} - \eta_t \Gamma_{c,t} + \eta_t^s \frac{\sigma_t}{c_t}\end{aligned}$$

The first order condition for the clean good in Appendix B.1 gives

$$\frac{\phi_t}{f_t} = \frac{1-\widetilde{\chi}_t}{U_{c,t}}$$

At the full optimum, the restriction terms disappear and $\widetilde{\chi}_t = \chi_t$. For $t < T$, the condition for w_t and integration of next period's costate condition give

$$\lambda_{1,t+1} = \beta R \frac{\phi_t}{f_t} = \mathbb{E}_t\left[\frac{\phi_{t+1}}{f_{t+1}}\right]$$

The second equality uses the endpoint and integrability conditions in Assumption 3.1 for the constrained multipliers, $\bar{\alpha}_{t+1} = 1$, and $\int_0^\infty f_{2,t+1}\, d\theta_{t+1} = 0$, as in Appendix A.4. Substituting the first order condition for the clean good gives

$$\frac{1-\widetilde{\chi}_t}{U_{c,t}} = \frac{1}{\beta R}\,\mathbb{E}_t\left[\frac{1-\widetilde{\chi}_{t+1}}{U_{c,t+1}}\right] \tag{28}$$

If we define $\mathcal{G}_t \equiv 1 - \gamma_{c,t}\frac{\psi U_{c,t}}{\theta f_t} + \eta_t \Gamma_{c,t}$,

$$1 - \widetilde{\chi}_t = \mathcal{G}_t - \eta_t^s \frac{\sigma_t}{c_t} \tag{29}$$

Suppose $\mathcal{G}_t > 0$ at every date and $\mathbb{E}_t[\mathcal{G}_{t+1}] < \infty$ for $t < T$. Define $\tau_t^{s,*}$ by $1 - \tau_t^{s,*} = \mathcal{G}_t / \mathbb{E}_t[\mathcal{G}_{t+1}]$. To study the timing of the savings multiplier, I rewrite the inverse Euler condition as a relation between $\eta_t^s \sigma_t / c_t$ and its expected value next period. The covariance term shows why the gap $\bar{\tau}_t^s - \tau_t^{s,*}$ alone does not determine this movement under uncertainty. Multiplying (28) by $U_{c,t}$ and substituting the Euler equation with the fixed savings wedge gives

$$
\begin{aligned}
1-\widetilde{\chi}_t &= \frac{U_{c,t}}{\beta R}\,\mathbb{E}_t\left[\frac{1-\widetilde{\chi}_{t+1}}{U_{c,t+1}}\right] \\
&= (1-\bar{\tau}_t^s)\,\mathbb{E}_t[U_{c,t+1}]\,\mathbb{E}_t\left[\frac{1-\widetilde{\chi}_{t+1}}{U_{c,t+1}}\right] \\
&= (1-\bar{\tau}_t^s)\left\{\mathbb{E}_t[1-\widetilde{\chi}_{t+1}] - \mathrm{Cov}_t\left(U_{c,t+1}, \frac{1-\widetilde{\chi}_{t+1}}{U_{c,t+1}}\right)\right\}
\end{aligned}
$$

When the displayed conditional moments are finite, the last equality follows from the definition of conditional covariance and $U_{c,t+1}(1-\widetilde{\chi}_{t+1})/U_{c,t+1} = 1-\widetilde{\chi}_{t+1}$. The covariance term measures how future marginal utility varies with the marginal resource cost of delivering utility, conditional on today's history. Substitution of (29) then yields

$$
\eta_t^s \frac{\sigma_t}{c_t} - (1-\bar{\tau}_t^s)\,\mathbb{E}_t\left[\eta_{t+1}^s \frac{\sigma_{t+1}}{c_{t+1}}\right] = \mathbb{E}_t[\mathcal{G}_{t+1}](\bar{\tau}_t^s - \tau_t^{s,*}) + (1-\bar{\tau}_t^s)\,\mathrm{Cov}_t\left(U_{c,t+1}, \frac{1-\widetilde{\chi}_{t+1}}{U_{c,t+1}}\right) \quad (30)
$$

Let $z_t = \eta_t^s \sigma_t / c_t$. For $T \geq 1$, suppose $\mathbb{E}_t[|z_{t+1}|] < \infty$ and suppose (13) holds almost surely at every $t < T$. Equation (30) then has a strictly positive right-hand side. Set $x_{T+1} = 0$. Equation (12) and the definition of σ_t give, for $t = 0, \ldots, T$,

$$
\begin{aligned}
\eta_t^s &= P_t U_{c,t}(x_t - x_{t+1}) \\
z_t &= -P_t U_{cc,t}(x_t - x_{t+1})
\end{aligned}
$$

x_{t+1} is known at date t and $-P_t U_{cc,t} \geq 0$ by Assumption 2.2. By backward induction, almost surely,

$$
x_{t+1} < \max\{x_t, 0\} \qquad t < T
$$

At $t = T-1$, if $x_T \geq \max\{x_{T-1}, 0\}$, then $z_{T-1} \leq 0$ and $z_T \geq 0$, contradicting (13). At an earlier date, suppose the bound holds at $t+1$. On any date-t event where $x_{t+1} \geq \max\{x_t, 0\}$, we have $z_t \leq 0$, while the induction hypothesis gives $x_{t+2} < x_{t+1}$ and hence $z_{t+1} \geq 0$ almost surely. The conditional expectation of z_{t+1} is therefore nonnegative on that event, again contradicting (13). The event must have probability zero.

Starting from $x_0 = 0$, the bound gives $x_t < 0$ for $t = 1, \ldots, T$. The first identity then gives $\eta_0^s > 0$ and $\eta_T^s < 0$. Finally, (26) gives

$$\sum_{j=0}^{t} \frac{\eta_j^s}{P_j U_{c,j}} = x_0 - x_{t+1} = -x_{t+1} > 0 \qquad t < T$$

Applying the same argument to $-x_t$ and $-z_t$ proves the negative case.

For the following sign-order result, suppose $U_{c,t+1}$, $\mathcal{G}_{t+1}$, and z_{t+1} are known conditional on today's history at every $t < T$. The covariance disappears and the conditional expectations equal the realized values, giving

$$\eta_t^s \frac{\sigma_t}{c_t} - (1 - \bar{\tau}_t^s)\eta_{t+1}^s \frac{\sigma_{t+1}}{c_{t+1}} = \mathcal{G}_{t+1}(\bar{\tau}_t^s - \tau_t^{s,*}) \tag{31}$$

Assume $\sigma_t > 0$ at every date and $1 - \bar{\tau}_t^s > 0$ for $t < T$. Strict curvature is needed because Assumption 2.2 alone allows $\sigma_t = 0$. Equation (31) reads

$$z_t = (1 - \bar{\tau}_t^s) z_{t+1} + \mathcal{G}_{t+1}(\bar{\tau}_t^s - \tau_t^{s,*})$$

For $T \geq 1$, suppose $\bar{\tau}_t^s > \tau_t^{s,*}$ for all $t < T$. The recursion implies that $z_t \leq 0$ entails $z_{t+1} < 0$, and the strict wedge gaps rule out an identically zero sequence. Equation (27) gives

$$\sum_{t=0}^{T} \frac{c_t}{\sigma_t P_t U_{c,t}} z_t = 0$$

All weights are positive, so both signs must appear. Once z_t becomes nonpositive, every subsequent value is strictly negative. The sequence therefore starts positive and ends negative, with one sign change and at most one intervening zero. Since $\sigma_t / c_t > 0$, the same sign ordering holds for η_t^s. If $\bar{\tau}_t^s < \tau_t^{s,*}$ for every $t < T$, the ordering reverses.

C Derivations with age-dependent and linear carbon taxes

C.1 Expenditure, demand, and incentive compatibility

To simplify notation, let $\bar{p}_t \equiv 1 + \bar{\tau}_t^q$. The planner has this consumption wedge constraint:

$$\Gamma_t^q \equiv \frac{U_{q,t}}{U_{c,t}} - \bar{p}_t = 0$$

At fixed price, labor, and pollution, define indirect utility by

$$\tilde{U}(x, l; \bar{p}, S) = \max_{c,q} \; U(c, q, l, S) \quad \text{s.t.} \quad c + \bar{p}q = x$$

x is the expenditure on the two goods. If we substitute $c = x - \bar{p}q$, we get the first order condition $U_q = \bar{p}U_c$. At an optimum with $\bar{p} > 0$, expenditure and labor determine the consumption mix at the common price. Let $c^d(x, l; \bar{p}, S)$ and $q^d(x, l; \bar{p}, S)$ denote conditional demands. At fixed price and pollution, differentiating the expenditure constraint gives

$$\begin{aligned} \frac{\partial c^d}{\partial x} + \bar{p}\frac{\partial q^d}{\partial x} &= 1 \\ \frac{\partial c^d}{\partial l} + \bar{p}\frac{\partial q^d}{\partial l} &= 0 \end{aligned}$$

Using $U_q = \bar{p}U_c$, the derivatives of indirect utility are

$$\begin{aligned} \tilde{U}_x &= U_c\frac{\partial c^d}{\partial x} + U_q\frac{\partial q^d}{\partial x} = U_c\left(\frac{\partial c^d}{\partial x} + \bar{p}\frac{\partial q^d}{\partial x}\right) = U_c \\ \tilde{U}_l &= U_l + U_c\frac{\partial c^d}{\partial l} + U_q\frac{\partial q^d}{\partial l} = U_l + U_c\left(\frac{\partial c^d}{\partial l} + \bar{p}\frac{\partial q^d}{\partial l}\right) = U_l \end{aligned}$$

Holding expenditure, earnings, price, and pollution fixed, the direct effect of skill is

$$\frac{\partial}{\partial\theta}\tilde{U}\left(x, \frac{y}{\theta}; \bar{p}, S\right) = -\tilde{U}_l\frac{y}{\theta^2} = -U_l\frac{y}{\theta^2}$$

At a truthful allocation, the assigned bundle satisfies $U_q = \bar{p}U_c$ and coincides with conditional demand. These identities preserve the local incentive envelope in (7), including the continuation term.

Global incentive compatibility requires an additional comparison when households can reallocate expenditure after misreporting. The bundle assigned to report $\hat{\theta}$ remains feasible for a household with true skill θ and

$$\tilde{U}\left(x(\hat{\theta}), \frac{y(\hat{\theta})}{\theta}; \bar{p}_t, S_t\right) \geq U\left(c(\hat{\theta}), q(\hat{\theta}), \frac{y(\hat{\theta})}{\theta}, S_t\right)$$

This inequality compares two deviation payoffs. It can be strict because the deviating household supplies labor $y(\hat{\theta})/\theta$ rather than $y(\hat{\theta})/\hat{\theta}$, which can change its preferred consumption mix. In a one-period economy, incentive compatibility of the allocation of expenditure and earnings requires, for every θ and $\hat{\theta}$,

$$\tilde{U}\left(x(\theta), \frac{y(\theta)}{\theta}; \bar{p}, S\right) \geq \tilde{U}\left(x(\hat{\theta}), \frac{y(\hat{\theta})}{\theta}; \bar{p}, S\right)$$

In the dynamic problem, let $x_t(\theta^t) \equiv c_t(\theta^t) + \bar{p}_t q_t(\theta^t)$. For every initial skill θ and reporting strategy σ, incentive compatibility requires

$$\mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \tilde{U}\left(x_t(\theta^t), \frac{y_t(\theta^t)}{\theta_t}; \bar{p}_t, S_t\right) \middle| \theta\right] \geq \mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \tilde{U}\left(x_t(\sigma^t(\theta^t)), \frac{y_t(\sigma^t(\theta^t))}{\theta_t}; \bar{p}_t, S_t\right) \middle| \theta\right]$$

Here, $\sigma^t(\theta^t)$ is the reported history when the true history is θ^t and expectations are conditional on true initial skill θ. The following derivations characterize the local relaxation with a common consumption wedge.

Differentiate $U_q = \bar{p} U_c$, using the expenditure constraint. Define $\mathcal{D} \equiv U_{qq} - 2\bar{p}U_{cq} + \bar{p}^2 U_{cc}$. At fixed labor and pollution, expenditure and compensated price changes give

$$\frac{\partial q^d}{\partial x} = \frac{U_{cq} - \bar{p}U_{cc}}{|\mathcal{D}|} = \frac{\bar{p}U_c \mathcal{I}_c^q}{|\mathcal{D}|}$$
$$s_{qq} \equiv \left.\frac{\partial q^d}{\partial \bar{p}}\right|_u = \frac{U_c}{\mathcal{D}} < 0$$

Writing $\mathcal{I}_q^q \equiv \partial \ln(U_q/U_c)/\partial q$, these identities imply

$$\Delta_t^q \equiv \bar{p}\mathcal{I}_c^q - \mathcal{I}_q^q = -\frac{\mathcal{D}}{\bar{p}U_c} = \frac{1}{\bar{p}|s_{qq}|} > 0 \tag{32}$$

C.2 Optimal age-dependent consumption wedge

The Lagrangian of Appendix A.2 gains $\int \eta^q \Gamma^q dF^t$. Using the usual willingness-to-pay identities,

$$\Gamma^q_z = \frac{U_{qz} - \bar{p} U_{cz}}{U_c} = \bar{p}\, \mathcal{I}^q_z. \tag{33}$$

Each pointwise condition for c, q, and l gains $\eta^q f \Gamma^q_z$. With income wedges free to adjust,

$$\frac{\phi U_c}{f} = 1 - \gamma_c \frac{\psi U_c}{\theta f} + \eta^q \bar{p}\, \mathcal{I}^q_c \tag{34}$$

$$\frac{\phi U_c}{f} = \frac{1 + \nu_t}{\bar{p}} - \gamma_q \frac{\psi U_c}{\theta f} + \eta^q \mathcal{I}^q_q \tag{35}$$

Here ν_t is the social cost of carbon in (16). Equating (34) and (35) gives

$$\underbrace{1 - \frac{1 + \nu_t}{\bar{p}_t}}_{\varpi_t} = \underbrace{-(\gamma_q - \gamma_c)\frac{\psi U_c}{\theta f}}_{\varpi^*_t} - \eta^q_t \Delta^q_t$$

Using (32),

$$\eta^q_t = \bar{p}_t |s_{qq,t}| (\varpi^*_t - \varpi_t)$$

The common wedge enters the date-0 resource-cost Lagrangian only through the date-t price constraints. Accounting for resource discounting and transition probabilities gives

$$\frac{d\mathcal{L}_0}{d\bar{\tau}^q_t} = -\frac{1}{R^t}\, \mathbb{E}_0[\eta^q_t]$$

The planner minimizes resource cost at fixed promised welfare, so a negative derivative favors raising the consumption wedge. An interior optimum sets this derivative to zero. Since $\bar{p}_t$ and ϖ_t are common across date-t histories,

$$\varpi_t = \frac{\mathbb{E}_0[|s_{qq,t}| \varpi^*_t]}{\mathbb{E}_0[|s_{qq,t}|]}$$

Weak separability gives $\varpi^*_t = 0$ at every history, so the optimal date-specific rate is ν_t.

C.3 Fixed income wedges and an age-dependent consumption wedge

The conditions for c, q, and l gain $\eta^q f \Gamma_z^q$. Equating the resulting conditions for c and q,

$$\varpi_t = \varpi_t^* - \eta_t \left(\Gamma_c - \frac{\Gamma_q}{\bar{p}_t} \right) + \eta_t^s \mathcal{I}_{c,t}^q - \eta_t^q \Delta_t^q$$

Using the Γ definitions and rewriting as in Appendix B.1,

$$\Gamma_c - \frac{\Gamma_q}{\bar{p}_t} = (1 - \bar{\tau}_t^y) \mathcal{I}_{c,t}^q + \frac{\gamma_{q,t} - \gamma_{c,t}}{y_t}$$

Putting the $\mathcal{I}_{c,t}^q$ terms together,

$$\varpi_t = \varpi_t^* - \eta_t \frac{\gamma_{q,t} - \gamma_{c,t}}{y_t} - ((1 - \bar{\tau}_t^y)\eta_t - \eta_t^s) \mathcal{I}_{c,t}^q - \eta_t^q \Delta_t^q$$

Alternatively,

$$\varpi_t = \varpi_t^* - \eta_t \frac{\gamma_{q,t} - \gamma_{c,t}}{y_t} - \zeta_t \mathcal{I}_{c,t}^q - \eta_t^q \Delta_t^q$$

With labor and savings wedges fixed, the age-dependent consumption wedge satisfies

$$\varpi_t = \frac{\mathbb{E}_0[|s_{qq,t}| \ \varpi_t^{**}]}{\mathbb{E}_0[|s_{qq,t}|]}$$

For a linear carbon tax, the consumption wedge is constant across households and ages. Summing across dates and substituting the multiplier formula gives

$$\sum_{t=0}^{T} \frac{1}{R^t} \mathbb{E}_0[|s_{qq,t}|(\varpi_t^{**} - \varpi_t)] = 0$$

For a time-varying social cost of carbon, $\varpi_t = (\bar{\tau}^q - \nu_t)/\bar{p}$ and this is equivalent to

$$\bar{\tau}^q = \frac{\sum_{t=0}^{T} \frac{1}{R^t} \mathbb{E}_0[|s_{qq,t}|(\nu_t + \bar{p}\varpi_t^{**})]}{\sum_{t=0}^{T} \frac{1}{R^t} \mathbb{E}_0[|s_{qq,t}|]}$$

C.4 Social cost of carbon

The requirement that the consumption wedge be common across households at each date affects the SCC adjustment factor through both the direct pollution derivative and the delivery cost ϕ. From (33) and (34), their combined contribution at each history is

$$\eta^q \left(\Gamma_S^q - \Gamma_c^q \frac{U_S}{U_c} \right) = -\eta^q \frac{U_S}{U_c} \left(\bar{p} \mathcal{I}_c^S - \mathcal{I}_q^S \right)$$

This follows directly from $\mathcal{I}_z^S = \frac{\partial \ln(-U_S/U_c)}{\partial z}$ and the quotient rule. With income wedges free to adjust, substituting into the condition of Appendix A.6 yields

$$\mu_t - \frac{1}{R} \frac{\partial V_{t+1}}{\partial S_t} = -\mathbb{E}_0 \left[\left(1 + (\xi - \gamma_c) \frac{\psi U_c}{\theta f} + \eta^q \left(\bar{p} \mathcal{I}_c^S - \mathcal{I}_q^S \right) \right) \frac{U_S}{U_c} \right]$$

The expectation averages all date-t histories. With income wedges fixed, this adds the same η^q term to the bracket of (25). The recursion for ν_t (16) is unchanged, so aggregating the current-date costs on the right-hand side as in Appendix A.6 gives the social cost of carbon.

Choosing the common wedge optimally does not generally remove the additional term. At a date whose price is free to adjust, $\mathbb{E}_0[\eta_t^q] = 0$ and its contribution to the social cost of carbon is

$$\mathbb{E}_0[\eta_t^q \mathcal{K}_t] = \mathrm{Cov}_0(\eta_t^q, \mathcal{K}_t) \qquad \mathcal{K}_t \equiv -\frac{U_{S,t}}{U_{c,t}} \left(\bar{p}_t \mathcal{I}_{c,t}^S - \mathcal{I}_{q,t}^S \right)$$

The social cost remains common across households because pollution is aggregate, but its level can change with the constrained allocation.